A Low Complex Algorithm For Interpolation as well as Lossless Compression of Natural Images

*Nimisha Agarwal, *Ayush Kumar, *Juhi Bhadviya, †Giovanni Ramponi

*The LNM Institute of Information Technology, Jaipur, India
†University of Trieste, Trieste, Italy
Email: * nimisha.agarwal28@gmail.com, *ayushkd15@gmail.com,*juhi689@gmail.com, †ramponi@units.it

***Abstract*—This paper presents a new generic algorithm for image interpolation as well as lossless image coding. Main motivation behind the work is to reduce computational complexity involved in using Least Square Error Minimization (LS). The proposed method down samples the given image to its quarter size and then to its (1/16)th size. For each downsampled image, the least Square predictors are then obtained corresponding to pixels belonging to each bin. Thus, these predictors are used to synthetically generate a set of optimal predictors corresponding to each bin of the original image. Our proposed algorithm thus reduces 60% to 70% of computational complexity. We also observed that proposed algorithm gives insignificant loss in terms of compression ratio as compared with some of the previous works reported in literature.**

***Index Terms*—Image Interpolation, Optimal Predictors, Context-Based, Least Square, Compression and Computational Complexity.**

I. Introduction

Image interpolation is a technique of finding unknown pixels in a high resolution image with the help of known pixels of the corresponding low resolution image. It has many applications in the field of medical imaging, computer vision, remote sensing, satellite imaging and many more.

Li and Orchard [1] deviced an interpolation method which estimated co-variance corresponding to pixels of a low resolution image and the missing pixels are then interpolated based on this co-variance (NEDI) [1]. Jakhetiya and Tiwari [2] had proposed a block based interpolation technique SPIA (single pass Interpolation algorithm). Jakhetiya and Jaiswal had proposed a computationally efficient context based interpolation [3] algorithm. Jakhetiya and Jaiswal [4] deviced a switching based technique for interpolation based on Soft Adaptive Interpolation (SAI) and Single Pass Interpolation Algorithm (SPIA). All these interpolation algorithms are based on Least Square Error Estimation and are used to preserve edge structures. Hence, these require high computational power.

On the other hand, image compression is a technique of reducing the redundancy of data in order to transmit the image data effectively and is also used for storage purposes. Compression is used for some specific information extraction, medical imaging, in scientific and industrial sector etc.

Li and Orchard had proposed an Edge Directed Prediction(EDP) [5] algorithm in which LS based optimization done is applied only on the edged pixels. Kau and Lin had proposed Run-length and Adaptive Linear Predictor [6] RALP coding scheme in which LS based optimisation is done only when an edge is detected or when predictor error is greater than the predefined threshold. Tiwari and Kumar [7] had proposed a Context-Based Image Compression Algorithm which finds switched adaptive LS based parameters for pixels belonging to various slope bins of GAP [8]. Jakhetiya and Jaiwal [9] deviced a lossless coding scheme based on similar structure as that of interpolation. Thus, it can be seen that to preserve edge structure high complex Least Square Based methods are used.

A new prediction scheme for both interpolation and lossless compression of images is proposed. The main incentive of this proposed algorithm is to show significantly lower complexity than some other solutions proposed in literature. It works as follows:

1) We downsample the given image to $(1/4)^{th}$ of its original size and then, classified the existing pixels of the given image into different classes called bins.
2) Predictor scheme [3],[7] was used, to get switched optimal predictors (LS Based) for pixels belonging to each bin.
3) We then performed a simple experiment, i.e, obtained predictors of each bin are applied to pixels of corresponding bin of the original image, but it results into poor quality.
4) Thus, we further went for downsampling the given image to $(1/16)^{th}$ size and again used prediction scheme [3],[7] and obtained switched optimal predictors of each bin.
5) With the help of two set of parameters for each bin, we generate a new set of parameters for pixels of each bin which results into insignificant loss in quality.
6) Thus, it reduces complexity to a great extent as compared to recent works in literature.

This paper is further classified in various sections. Section II describes the already existing algorithms. Section III elaborates the proposed algorithm for both interpolation and lossless compression. Section IV describes the computational complexity analysis. Sections V and VI futher presents the simulation results and conclusions respectively.

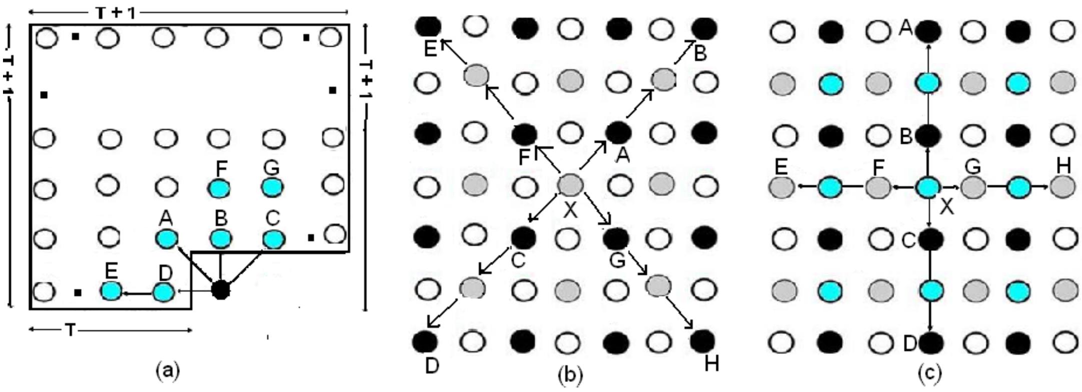

Fig. 1. (a) Training window used to optimize the prediction coefficients in EDP, RALP and [7], (b) and (c) are two phase predictor structure used in [3]

II. Review of existing method

A. Context Based Lossless Image Compression Algorithm [7]

This method classifies the existing pixels of a given image into seven classes and then finds Switched Optimal Predictor using LS based predictor method for each class as shown in Fig. 1 (a). Classification of pixels depending upon the slope (S) value is shown in (1).

$$S = d_v - d_h \tag{1}$$

where $d_h = |D - E| + |B - A| + |B - C|$ and
$d_v = |A - D| + |B - F| + |C - G|$

The slope bin boundaries are as follows: $S > 80$, $S < -80$, $32 < S \leq 80$, $8 < S \leq 32$, $-80 \leq S < -32$, $-32 \leq S < -8$ and $-8 \leq S \leq 8$.

B. Context Based Image Interpolation Algorithm [3]

The proposed method classifies the existing pixels of a given image into 8 classes in two different phases. Then, it finds Optimal Predictors using LS based Predictor method for each class of both the phase as shown in Fig. 1 (b), (c). First phase includes prediction of only even-even pixels whereas, second phase includes prediction of the remaining missing pixels. In first phase, slope (S1) value for classification is shown in (2).

$$S1 = d_{45} - d_{135} \tag{2}$$

where $d_{135} = |E - F| + |F - G| + |G - H|$ and
$d_{45} = |C - D| + |C - A| + |A - B|$

In the second phase, slope (S2) value for classification is shown in (3).

$$S2 = d_v - d_h \tag{3}$$

where $d_h = |E - F| + |F - G| + |G - H|$ and
$d_v = |A - B| + |B - C| + |C - D|$

Slope Bin boundaries for both the phase are same and they are as follows: $S > 40$, $S < -40$, $20 < S \leq 40$, $-40 < S \leq -20$, $-20 \leq S < -8$, $0 \leq S < 8$ and $8 \leq S \leq 20$.

III. Proposed Self Evaluated Optimal Parameters For Prediction

The proposed algorithm is a low complex algorithm for image interpolation and lossless image coding. Both these proposed algorithms are discussed seperately in subsequent sub-sections.

A. Proposed Low Complex Interpolation Algorithm

Suppose we are originaly given an LR image of dimensions 256×256. We now downsample the given LR image and get a low resolution LLR image (128×128) using (4).

$$LLR\,(i,j) = LR\,(2i-1, 2j-1) \quad \forall (i,j) \in LLR \tag{4}$$

The further process works as follows:

1) We classified the pixels of LLR image in a total of 16 bins using Context-Based Image Interpolation algorithm [7] as indicated in section II(B), such that similar kind of pixels belong to the same bin.
2) Then, we obtained fourth order LS optimal predictor for each bin of LLR (128×128) image.
3) As a result we get an image of higher resolution (256×256).
4) The LS parameter values of m^{th} bin in LLR image can be denoted as E_{128}^{m}(i) where i varies from 1 to 4 and m ranges from 1 to 16 (Total no. of bins).

To make this algorithm computationally cheaper, we directly applied these LS based parameters (E_{128}^{m}(i)) to corresponding pixels of m^{th} bin of the original LR image. Thus, we obtained an image of higher dimensions 512×512, without evaluating its original parameters. But, we failed to achieve better prediction accuracy because the edges in LLR image are more concentrated as compared to LR image. Hence, optimal predictors E_{128}^{m}(i) alone cannot be applied directly to 256×256. To justify this, Mean Square Error (MSE) is obtained for interpolated image (512×512) using E_{128}^{m} and using original LS parameters (Original LS parameters can be obtained by applying method [3] to LR image). From Table I, we can conclude that MSE obtained using original parameters

TABLE I
MEAN SQUARE ERROR(MSE) VALUES OBTAINED USING ORIGINAL(O) PARAMETER, DOWNSAMPLED(D) PARAMETER AND PROPOSED(P) PARAMETERS

Images	MSE(O)	MSE(D)	MSE(P)
1.	265.5	304.3	267.96
2.	58.35	85.21	58.62
3.	74.48	108.1	73.97

(MSE(O)) is less than the MSE obtained using parameters ((MSE(D)) of downsampled images.

Thus, directly using E_{128}^{m}(i) parameters for interpolating LR image results into a poor quality of image, although it reduces computational complexity about $(1/4)^{th}$ times of the original amount. Hence, we further downsampled the LR image to $(1/16)^{th}$ of its size and it works as follows:

1) Our new downsampled L_LR image (64×64) can be obtained using (5).
$$L\_LR\,(i,j) = LR\,(4i-3, 4j-3) \quad \forall (i,j) \in L\_LR \quad (5)$$
2) Pixels of this image are again classified in a total of 16 bins and context based interpolation algorithm [3] is applied, to obtain LS based predictors and hence, get a higher resolution image of size 128×128.
3) The computed LS optimal parameters for mth bin of L_LR image can be denoted by E_{64}^{m}(i) where i varies from 1 to 4 and m ranges from 1 to 16.

We still failed to achieve better prediction accuracy if these LS based predictors of m^{th} bin of L_LR image are directly applied to m^{th} bin of the original image. However, computational conplexity reduces by $(1/16)^{th}$ times.

Thus, after getting parameters for each bin of LLR and L_LR images respectively (E_{128}^{m}(i), E_{64}^{m}(i)), we make best use of them to generate a new set of parameters that can be applied to LR image.

B. Efficient Generation Of Optimal Predictor

Let the fourth order LS based predictor for a particular bin obtained using context based interpolation [3] of LR image, LLR image and L_LR image be a_i, b_i and c_i respectively where i varies from 1 to 4. Each of the parameter set for a particular bin follows: $a_1+a_2+a_3+a_4 = 1$, $b_1+b_2+b_3+b_4 = 1$ and $c_1 + c_2 + c_3 + c_4 = 1$.

With the help of b_i and c_i,we significantly generated a new parameter set d_i such that
$$d_i = (A_i \times b_i + B_i \times c_i) \quad (6)$$
where A_i and B_i are constants.

This is a generalized result and for one parameter (6) becomes, i.e d_1 can be represented as:
$$d_1 = (A_1 \times b_1 + B_1 \times c_1) \quad (7)$$
Now, the error generated between the original parameters a_i and the proposed parameters d_i is:
$$error = a_i - d_i = a_i - (A_i \times b_i + B_i \times c_i) \quad (8)$$
This error can be minimized using least square error minimization with respect to A_i and B_i and we found a covariance matrix as shown below.
$$\begin{bmatrix} \sum b_i^2 & \sum b_i c_i \\ \sum b_i c_i & \sum a_i^2 \end{bmatrix} \begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} \sum a_i b_i \\ \sum a_i c_i \end{bmatrix}$$
From this covariance matrix, we can estimate the values of A_i and B_i. We had done experiments on a large set of test images and found that the values of A_i in most cases varies from 0.639 to 0.665 and B_i varies from 0.342 to 0.3592. So, we kept the values of A_i and B_i to be 0.65 and 0.35 respectively and found that the loss (interpolation quality) in the performance is negligible as compared to the original predictors.

Thus, our new parameter d_i using b_i and c_i is found to be:
$$d_i = 0.65 b_i + 0.35 c_i \quad (9)$$
Our proposed interpolation algorithm works as follows:

1) Downsample the given LR (256×256) image to LLR image of dimensions 128×128 and find the LS optimal predictor values corresponding to each bin i.e E_{128}^{m}(i).
2) Again downsample the given LR image (256×256) to L_LR of dimensions 64×64 and find the LS optimal predictor values corresponding to each bin i.e E_{64}^{m}(i).
3) Use relation (9), to find the new predictor value set using E_{128}^{m} and E_{64}^{m}.
4) Thus repeat above steps and get parameters set for each bin of LR image. Then apply this new set of parameters to it, to interpolate LR image to get a higher resolution image of dimensions 512×512.

Thus, the proposed interpolation algorithm had found optimal predictor values for LR image using its downsampled image and has reduced the computational complexity to a greater extent without any significant loss in the quality. We will come to complexity analysis in section IV.

C. Proposed Lossless Image Compression Algorithm

The proposed context based compression algorithm remains almost same as in the previous sub-section. The only difference is that the predictor structure remains different in both the cases as shown in Fig. 1. Therefore it can be summarized as follows:

1) Suppose we are given an image I of dimensions 512×512.
2) Downsample the given image to a low resolution (LR) image of dimension 256×256 as shown in (10).
$$LR\,(i,j) = I\,(2i-1, 2j-1) \quad \forall (i,j) \in LR \quad (10)$$
3) We classified the existing pixels of the given LR image in a total of 7 bins according to context based compression algorithm as indicated in section II (A).
4) Then, we find the LS optimal predictors for pixels of each bin [7] and denote the parameters corresponding to m^{th} bin as E_{256}^{m}(i), where i varies from 1 to 4 and m varies from 1 to 7.

TABLE II
PERFORMANCE OF PROPOSED ALGORITHM (PRO_I, PRO_C AND PRO_FINAL) TERMS OF PSNR AND BIT PER PIXEL (BPP)

Interpolation (PSNR)					Compression (Zero Order Entropy)				Compression (Final Entropy)		
Images	NEDI	SPIA	[3]	Pro_I	EDP	RALP	[7]	Pro_C	CALIC	JPEG_LS	Pro_Final
1	23.82	23.79	**23.89**	23.85	4.61	**4.59**	4.62	4.63	4.56	4.86	**4.45**
2	30.37	30.41	30.47	**30.49**	4.78	**4.73**	4.80	4.83	4.58	4.71	**4.50**
3	29.21	29.37	29.41	**29.44**	4.40	**4.38**	4.43	4.41	**3.58**	3.84	3.65
4	30.84	30.89	30.94	**30.98**	4.66	4.66	**4.60**	4.62	4.06	4.27	**4.01**
5	33.38	33.43	**33.48**	33.41	4.58	4.59	4.56	**4.55**	3.47	4.19	**3.39**
6	27.36	27.38	27.44	**27.46**	5.00	5.01	**4.96**	5.01	4.21	4.45	**4.15**
7	21.56	21.50	**21.61**	21.59	5.71	**5.68**	5.76	5.72	**4.26**	5.16	4.67
8	30.87	**30.95**	30.91	30.93	4.33	4.35	4.28	**4.26**	3.39	3.53	**3.21**
9	27.14	**27.20**	27.18	27.16	5.06	5.02	4.99	**4.97**	4.15	4.33	**4.06**
10	30.60	**30.64**	30.58	30.55	4.14	4.12	4.12	**4.03**	4.06	**3.65**	3.78

5) We then performed the same experiment and applied this parameter set to the original image I.
6) We observed that applying these parameters directly to 512×512 image I results into poor performance.
7) So, we further downsampled the image to $(1/16)^{th}$ of its size to get a low resolution image LLR of size 128×128 as shown in (11).

$$LLR\,(i,j) = I\,(4i-3, 4j-3) \quad \forall (i,j) \in LLR \quad (11)$$

8) Again classifying the pixels into 7 different bins, we find the LS optimal predictor for each bin [7] denoted as E^{m}_{128}(i) for this LLR image (128 × 128).

Thus, again with the help of parameters of these two images, we try to find a new set of parameters d_i. We minimized the error generated between original parameters a_i and the proposed parameters d_i where $d_i = A_i \times b_i + B_i \times c_i$ as discussed in the previous subsection. We have done experiments on large set of images and found that if A_i and B_i is kept as 0.7 and 0.30 respectively, then loss in terms of Compression Ratio is negligible.

Thus, our new parameter d_i using b_i and c_i for lossless compression is found to be:

$$d_i = 0.7b_i + 0.3c_i \quad (12)$$

where b_i and c_i denotes parameters of LR and LLR images respectively.

We then apply this new set of parameters to original image of dimensions 512×512 and we are able to retrieve the original image (512 × 512) at the decoder end. Thus, we generated a lossless compression technique by applying a new set of parameters to the original image (512 × 512) to get negligible loss in terms of compression ratio along with reducing computational cost of the algorithm.

Hence, our proposed algorithm is a generic algorithm which reduces the computational complexity in both interpolation as well as lossless compression to a large extent without any significant loss in the quality of an image.

IV. COMPUTATIONAL COMPLEXITY

The proposed algorithm is a generic algorithm and requires less computational power as compared to existing LS based method. Complexity analysis is as follows:

A. Complexity Analysis With Interpolation Algorithm

Various interpolation methods reported in literature for edge preservation includes a high computational complexity associated with it. Suppose we have an image of dimension $A \times B$. Thus, the computational requirement of these methods are as follows:

1) NEDI [1] is a pixel by pixel interpolation method and it activates LS based method on some certain condition. Let it activates N_N number of LS based predictor and for each predictor it uses a block of 8x8 pixels. Thus, to interpolate the entire image, it requires $N_N \times (64 \times 20)$ number of multiplications, N_N matrix inversions and involve $N_N \times 64$ number of pixels.
2) SPIA [2] is a block (16×16) based interpolation algorithm. To interpolate each block of pixels, it involves 256×3 (256 pixels corresponding to fourth order and other pixels corresponding to two sixth order predictors), 3 matrix inversions of order 4 and 6, $(A \times B) \times 20/(16 \times 16)$ number of multiplications.
3) Method [3] uses context based interpolation algorithm in which LS based optimisation is done on 16 bins. Thus, it involves $A \times B$ number of pixels, $A \times B \times 20$ number of multiplications and 16 matrix inversions of order 4.
4) Whereas, the proposed algorithm involves only $(A \times B/4) + (A \times B/16)$ number of pixels, $(A \times B/4) + (A \times B/16) \times 20$ number of multiplications and 32 matrix inversions of order 4.

Thus, it can be seen that proposed algorithm has drasticaly reduced number of involvement of pixels, multiplications and matrix inversion.

B. Complexity Analysis with Compression algorithm

1) EDP and RALP estimates LS based predictors on some of the selected pixels and they select pixels for activation of LS adaptation. RALP activates LS adaptation on more number of pixels as compared to EDP. Let N_E and N_R be the number of pixels for EDP and RALP respectively.
2) To find LS based optimal predictor for a pixel, EDP involves $2T(T+1)$ number of pixels where $T = min(N, 7)$ and $N = 4$ for a fourth order predictor.
3) EDP involves $N_E \times 2T(T+1)$ multiplications and N_E matrix inversions, while RALP involves $N_R \times (2T(T+$

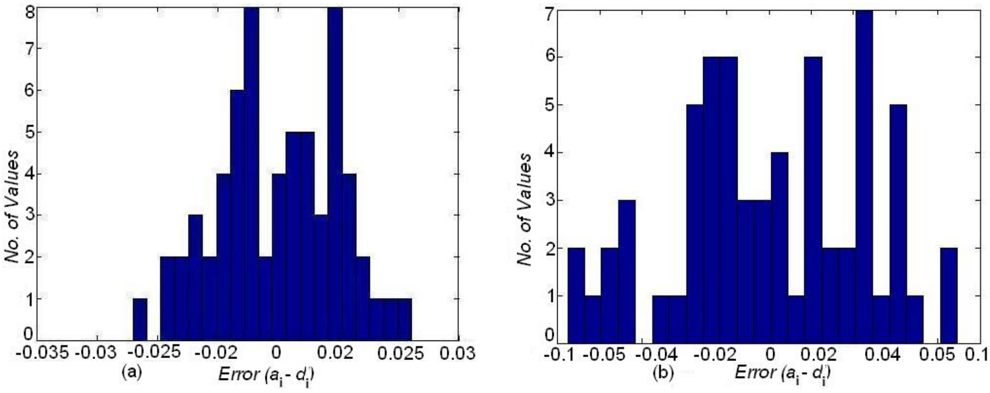

Fig. 2. Error estimation between original and proposed parameters (a) for interpolation and (b) for compression

1)) $\times 20$ multiplications and N_R matrix inversions where $N_R > N_E$.

4) Context-Based [7] method involves only $A \times B$ pixels, 14 matrix inversions and $A \times B \times 20$ number of multiplications.
5) Whereas, proposed algorithm involves $(A/2 \times B/2 + A/4 \times B/4)$ matrix inversions and $((A \times B)/4 + (A \times B)/16) \times 20$ multiplications.

Thus, our proposed algorithm has further reduced 60% to 70% involvement of pixels, multiplcation and reduces a lot of matrix inversions.

V. Simulation Results

The proposed algorithm is implemented and tested for a large set of test images [11] with varying characteristics for our simulation. A graph shown in Fig. 2 illustrates that the difference between original parameters and synthetically generated parameters is very small in many cases. Hence, proposed parameters along with saving a lot of computational complexity, produces insignificant loss in quality. We have made a complete lossless image coder and compared it with the JPEG_LS [10] and CALIC [8]. Ad-hoc context modeling has been used for predicted error image (residues). Bias cancellation methods have been used from CALIC. We can conclude from Table-2 that proposed algorithm has insignificant loss in terms of performance. From the Computational complexity analysis, we can say that it reduces a lot of computational power in both interpolation as well as in compression.

VI. Conclusions

In this paper, we proposed a computationally efficient algorithm for interpolation as well as lossless compression of natural images. In both cases, the proposed algorithm incorporates the relations of optimal predictor(using LS based method) of each bin of the two downsampled version and then apply it to the pixels belonging to the corresponding bin of the original image. By doing so, it involves less number of pixels and multiplications as compared to recent works in literature. Thus, the proposed self-evaluated switched optimal predictor scheme for both interpolation and lossless compression reduces a lot of computational power with insignificant loss in quality of images.

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