Homework #3

(Due: Nov 23)

Task 1. [100 Points] Randomized Multi-Pivot Quicksort

Consider a variant of randomized quicksort algorithm that always partitons around $k > 0$ pivots for some given integer constant k, and chooses the pivots as follows. It first selects ks numbers uniformly at random from the n input numbers, where $s = \frac{8\alpha}{(\alpha-1)^2} \ln n$, for some given $\alpha \in \left(0, \frac{k}{2}\right)$. It then sorts those ks numbers, and chooses every s-th element from the sorted sequence as a pivot.

(*a*) [**10 Points**] Give an efficient algorithm for partitioning n numbers around k pivots.

(*b*) [**50 Points**] Show that w.h.p. in n, no partition will contain more than $\frac{2\alpha n}{k}$ numbers.

(*c*) [**40 Points**] Give a high-probability (in n) bound on the running time of this variant of the quicksort algorithm.

Task 2. [100 Points] Jobs and Thieves

Suppose you have $m > 1$ servers and $n > 0$ jobs. Each job takes exactly one unit of time to execute, and initially all jobs are on one server. If a server has a list of jobs to execute, it executes them sequentially from the beginnning to the end of the list. Otherwise it selects another server uniformly at random, and attempts to steal half of that server's jobs (i.e., second half of the list). The steal attempt fails if the victim server does not have enough (i.e., more than one) jobs in its list. If multiple thieves attempt to steal from the same victim at exactly the same time, only one (can be anyone) of them succeeds. Each steal attempt (successful or not) takes exactly one unit of time to complete.

(*a*) [**40 Points**] Show that for any $\alpha > 1$, if you choose any subset S of $\alpha m \log m$ steal attempts from the set of all steal attemps in the entire system, then each of the m servers will be the victim of at least one steal attempt from S with probability at least $1 - \frac{1}{m^{\alpha-1}}$.

(*b*) [**30 Points**] Show that w.h.p. in m, after the initial $\Theta\left(m\left(\log m + \log\log n\right)\log n\right)$ steal attempts every steal attempt in the system will fail.

(*c*) [**30 Points**] Show that w.h.p. in m, all n jobs in the system will get executed in $\mathcal{O}\left(\frac{n}{m} + (m + \log n)\log\log n\right)$ time.