MIC-GPU: High-Performance Computing for Medical Imaging on Programmable Graphics Hardware (GPUs)

SPIE Medical Imaging

GPU-Acceleration of Individual Components

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Overview

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What to expect:

- details on the parallelization of various fundamental CT reconstruction algorithms
- details and fragment code for graphics-style GPU implementations of these
- insights on GPGPU-style implementations of these
- comparisons of results obtained with both

Decomposition

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$$P:\ p_i = \sum_{j=0}^{N^3-1} \left(v_j \cdot w_{ij} \right) \qquad B:\ v_j = \sum_{i=0}^{M_\varphi-1} \left(p_i \cdot w_{ij} \right)$$

FBP

$$v_j = \sum_{p_i \in P_{set}} p_i w_{ij\_fdk} = \boxed{\sum_{p_i \in P_{set}} B \cdot S}$$

S: scanner projections

I: identity projection/volume

Algebraic

$$v_j = v_j + \frac{\displaystyle\sum_{p_i \in P\varphi} \left(\frac{\lambda \left(p_i - \sum_{l=0}^{N^3-1} v_l \cdot w_{il} \right)}{\sum_{l=0}^{N^3-1} w_{il}} \right) w_{ij}}{\displaystyle\sum_{p_i \in P_\varphi} w_{ij}} = \boxed{v_j + \frac{B(\lambda \frac{S - P(V)}{P(I)})}{B(I)}}$$

OS-EM

$$v_j = \frac{v_j}{\sum_{p_i \in P_{set}} w_{ij}} \left(\sum_{p_i \in P_{set}} \left(\frac{p_i}{\sum_{l=0}^{N^3-1} v_l \cdot w_{il}} \right) w_{ij} \right) = \boxed{\frac{v_j}{\sum_{p_i \in P_{set}} B(I)} \left(\sum_{p_i \in P_{set}} B \frac{S}{P(V)} \right)}$$

Kernel-Centric Reconstruction

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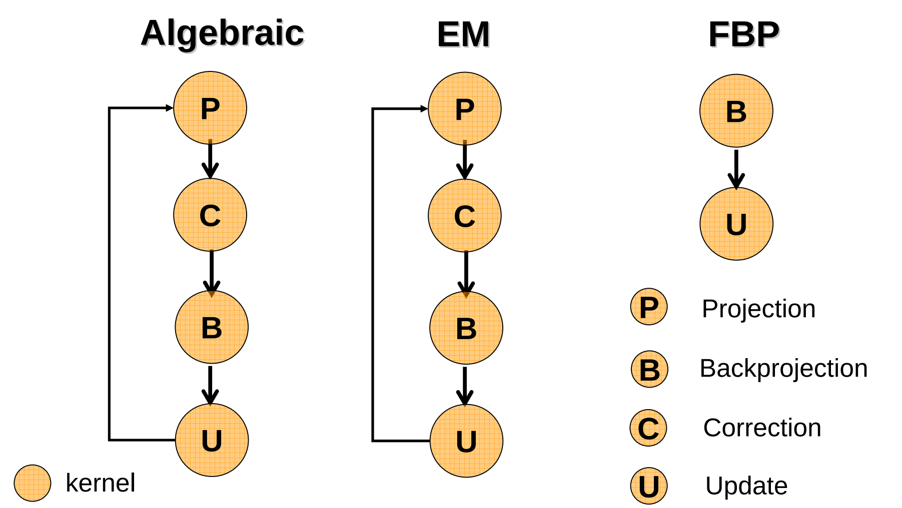
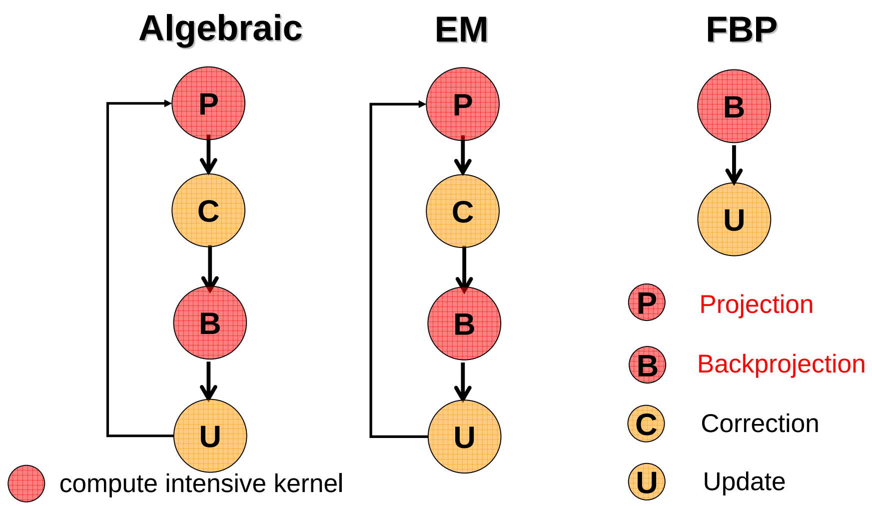
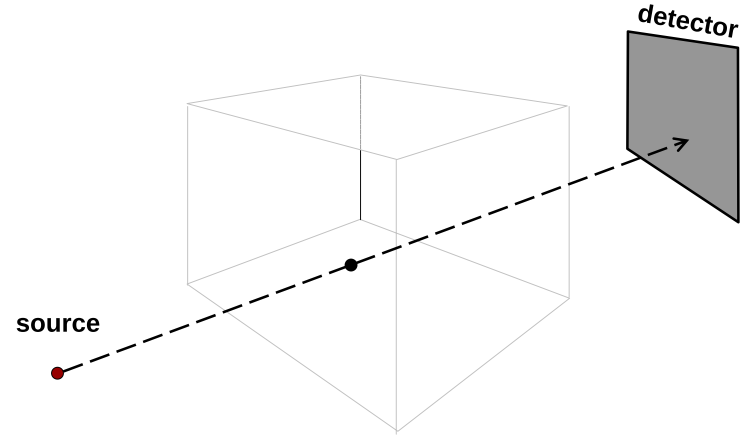


Kernel-Centric Reconstruction



Backprojection

Sample in projection space, voxel-driven

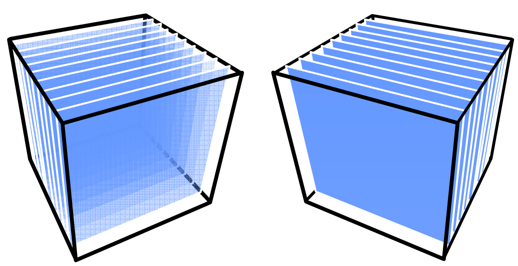


Volume Representation

3D texture is not used (no write support)

2D texture stacks are used

Axis aligned (x, y, z), easy to compute and store



Transformation Matrix

A 4x4 matrix *M* transforms 3D voxel coordinates to 2D pixel coordinates on the detector

Perform perspective divide if necessary (cone-beam)

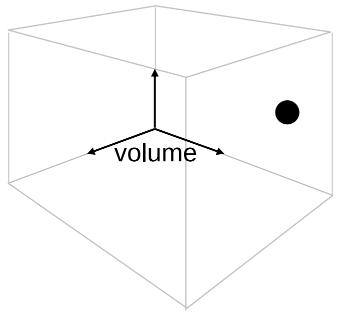
Composition of the matrix from graphics point of view

- model-view matrix
- projection matrix
- translation / scaling matrix

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = \begin{bmatrix} x_h \\ y_h \\ z_h \\ w_h \end{bmatrix} \qquad P_\varphi(U,V) = \left(\frac{x_h}{w_h}, \frac{y_h}{w_h}\right)$$

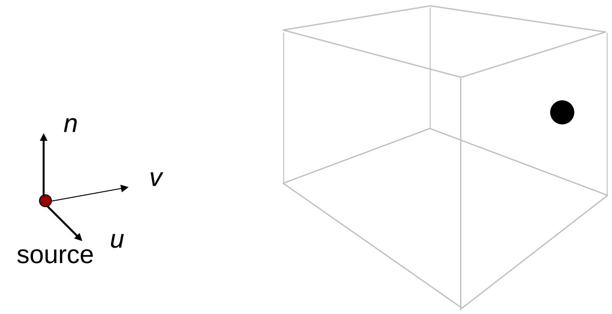
Decomposition

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



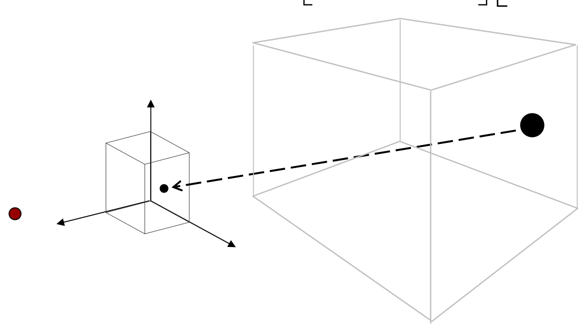
Decomposition: Model/View

$$\begin{bmatrix} u_x & u_y & u_z & -\vec{u}\cdot\vec{s} \\ v_x & v_y & v_z & -\vec{v}\cdot\vec{s} \\ n_x & n_y & n_z & -\vec{n}\cdot\vec{s} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



Decomposition: Projection

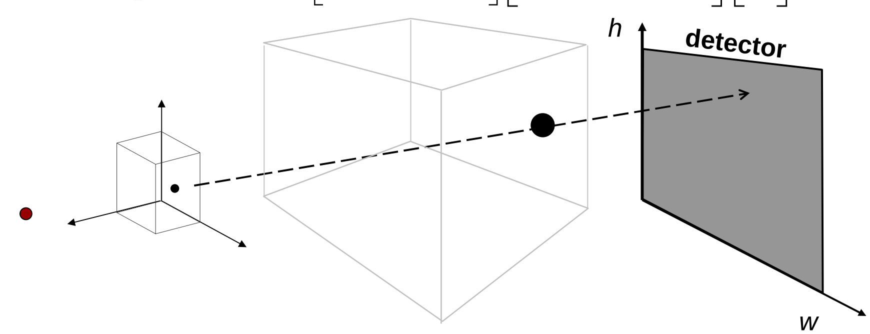
$$\begin{bmatrix} \frac{2n}{w} & 0 & 0 & 0 \\ 0 & \frac{2n}{h} & 0 & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{n-f} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z & -\vec{u}\cdot\vec{s} \\ v_x & v_y & v_z & -\vec{v}\cdot\vec{s} \\ n_x & n_y & n_z & -\vec{n}\cdot\vec{s} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



n,*f*: near/far viewing frustrum extent

Decomposition: Window

$$\begin{bmatrix} \frac{w}{2} & 0 & 0 & 0 \\ 0 & \frac{h}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1.0 \\ 0 & 1 & 0 & 1.0 \\ 0 & 0 & 1 & 1.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2n}{w} & 0 & 0 & 0 \\ 0 & \frac{2n}{h} & 0 & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{n-f} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z & -\vec{u}\cdot\vec{s} \\ v_x & v_y & v_z & -\vec{v}\cdot\vec{s} \\ n_x & n_y & n_z & -\vec{n}\cdot\vec{s} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



Full Transformation Matrix

Embedded transformation on graphics hardware (OpenGL)

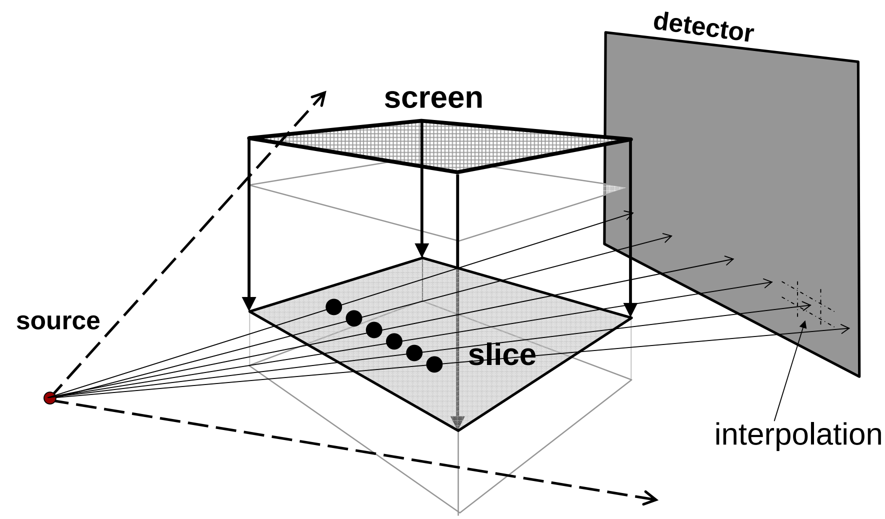
4th coordinate (w) contains voxel depth value

$$\begin{bmatrix} \frac{w}{2} & 0 & 0 & 0 \\ 0 & \frac{h}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1.0 \\ 0 & 1 & 0 & 1.0 \\ 0 & 0 & 1 & 1.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2n}{w} & 0 & 0 & 0 \\ 0 & \frac{2n}{h} & 0 & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{n-f} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z & -\vec{u}\cdot\vec{s} \\ v_x & v_y & v_z & -\vec{v}\cdot\vec{s} \\ n_x & n_y & n_z & -\vec{n}\cdot\vec{s} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = \begin{bmatrix} x_h \\ y_h \\ z_h \\ w_h \end{bmatrix}$$

$$w_h = n_x \cdot x_v + n_y \cdot y_v + n_z \cdot z_v - \vec{n}\cdot\vec{s} = \vec{n}\cdot\vec{v} - \vec{n}\cdot\vec{s}$$

$$|w_h| \Rightarrow \text{voxel depth}$$

Implementation



Implementation: GPGPU

Acquire transformation matrix *M*

Pass into fragment shader

Orthographically render the volume slice to be reconstructed

In the fragment shader, compute detector-plane coordinates:

- generate homogenous location coordinates for each fragment
- multiply by *M*

Then, also in the fragment shader:

- perform perspective-divide (*x*, *y*)
- extract *w* to compute depth weights
- interpolate densities (bilinear interpolation)

Fragment Code Sample

```
float  fVoxel = 0;
float4 vWorldPos = float4(wpos.x, y, wpos.y, 1); // voxel coordinates
for (int s = 0; s < iProjNo; s++){ // loop over projections
   // transform to detector positions, using 1st and 2nd matrix rows
   float2 vDetPos = float2(dot(vMatR0, vWorldPos), dot(vMatR1,vWorldPos));
   // compute depth values, using 3rd matrix row
   float   fDepth = dot(vMatR3, vWorldPos);
   vDetPos /= fDepth;              // perspective divide
   fWeight = pow(fSO/fDepth, 2);   // compute depth weights
   // sample+accumulate
   fVoxel += fWeight* tex3D(texProj3D, float3(vDetPos.x, vDetPos.y, idx+0.5));
}
```

Implementation: Graphics

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Vertex and fragment shader

Transformation in the vertex shader, much faster!

- acquire transformation matrix *M*
- pass into the vertex shader
- orthographic rendering of the slice to be reconstructed
- pass 3D coordinates of the vertices into the vertex shader
- in the vertex shader
  - multiply proxy polygon vertices coordinates with *M*

Fragments get rasterized

Only perform texture sampling in the fragment shader

- perspective-divide on each fragments, compute depth-weight
- interpolate densities (bilinear interpolation)

Code Sample

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Fragment code:

```
float  fDepthRCP = 1/vTransPos.w;            // reciprocal of voxel depth
float2 vDetPos = vTransPos.xy*fDepthRCP;     // perspective division
float  fWeight = pow(fSO*fDepthRCP, 2);      // compute voxel weights
float  fOldVal = texRECT(texSlice, wpos);    // sample original value
float  fNewVal = texRECT(texProj, vDetPos);  // sample incoming value
float  fOutVal += fOldVal + fNewVal*fWeight; // accumulate
return fOutVal;
```

GL code in host program:

```
glBegin(GL_POLYGON);
    glTexCoord3f(v00.x, v00.y, v00.z); glVertex3f(v00.x, v00.y, v00.z);
    glTexCoord3f(v10.x, v10.y, v10.z); glVertex3f(v10.x, v10.y, v10.z);
    glTexCoord3f(v11.x, v11.y, v11.z); glVertex3f(v11.x, v11.y, v11.z);
    glTexCoord3f(v01.x, v01.y, v01.z); glVertex3f(v01.x, v01.y, v01.z);
glEnd();
```


Graphics Pipeline Revisited

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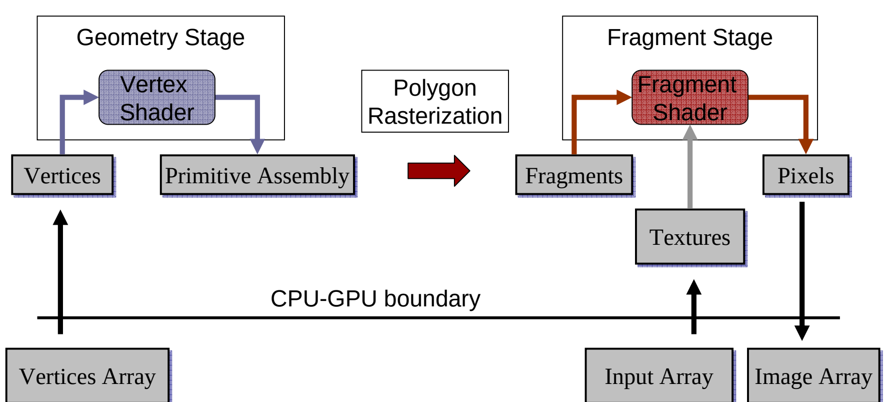

One pass

Pipeline 1: GPU as a Programmable Multi-Processor (MP-GPU)

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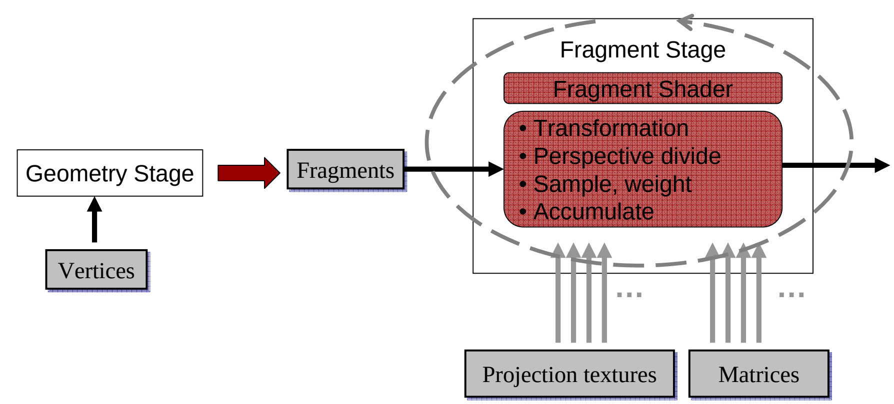
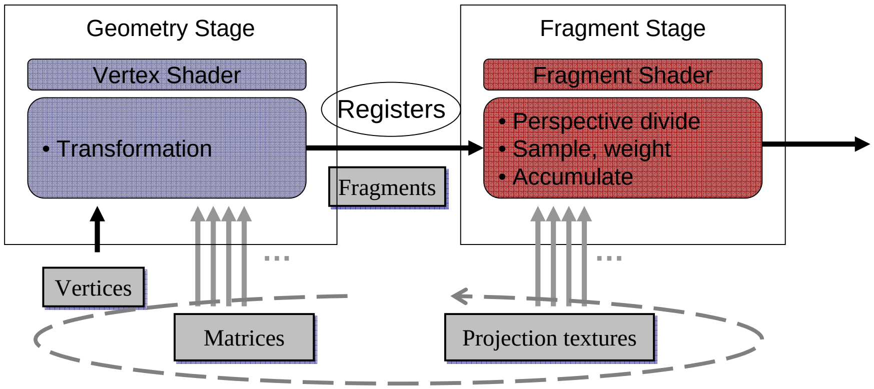


Fragments contain the (x,y,z) voxel coordinates

Pipeline 2: GPU as a Programmable Graphics Processor (AG-GPU)



Fragments contain the (u,v) detector space coordinates

Graphics Pipeline Benefits

Graphics-aware pipeline (AG-GPU) is considerably faster (~3×) than MP-GPU

- graphics facilities are hardwired!

There are further features that have their origins in graphics and come with GPUs:

- early fragment kill → eliminate fragments based on some condition before they even enter the fragment processor (AG-GPU+)
- hardwired 32-bit floating-point precision linear interpolations, matrix and vector arithmetic (+, -, *), frame-buffer blending and compositing
- RGBA parallelism

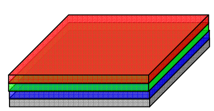
see Xu/Mueller, *Physics Medicine & Biology*, vol. 52, pp. 3405–3419, 2007

RGBA Parallelism

Exploit geometric mapping parallelism

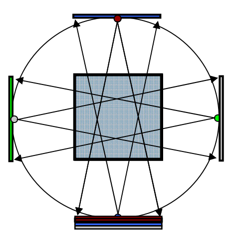
Volume packing

- adjacent 4 volume slices → RGBA



Projection packing

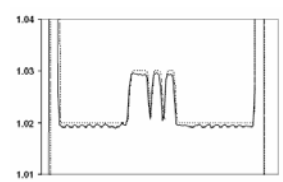
- symmetry in projection layout
- requires all projections beforehand



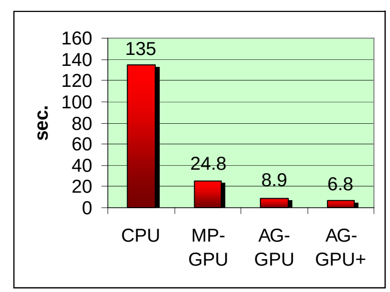
Example: Feldkamp Cone-Beam Reconstruction

360 projections (1024^2, general position), 512^3 volume

CPU GPU



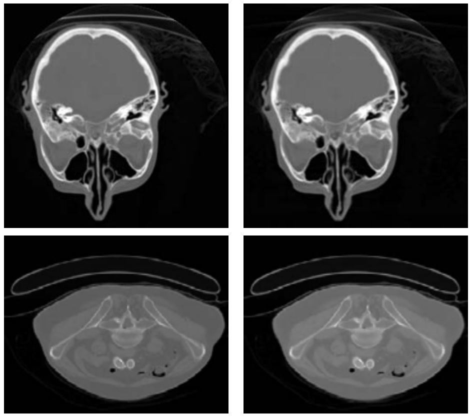
tumor profiles



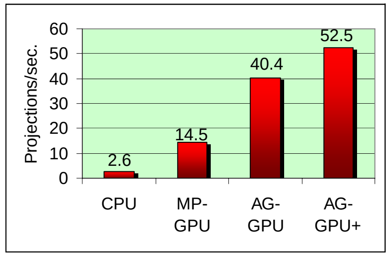
performance

Expressed in Projections/Sec.

360 projections, 512^3 volume



Original GPU-recon



performance in projections/s

GPU Enables Visual CT

High reconstruction frame rate enables injection of occasional volume rendering step

Also enables D^{2}VR: real-time volume visualization directly from projection data

see Xu/Mueller, *Proc. Volume Graphics Workshop*, pp. 23-30, 2006

Next: Iterative Algorithms

SART, EM

All require a projection simulation step

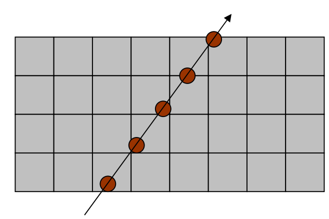
- should be as accurate as possible

Projection

Sampling in volume space, ray-driven

Raycasting methods [Krueger'03]

- represent volume as a single 3D texture
- rasterize the detector image, generate fragments
- cast rays, sample and accumulate
- no support for easy write



Projection

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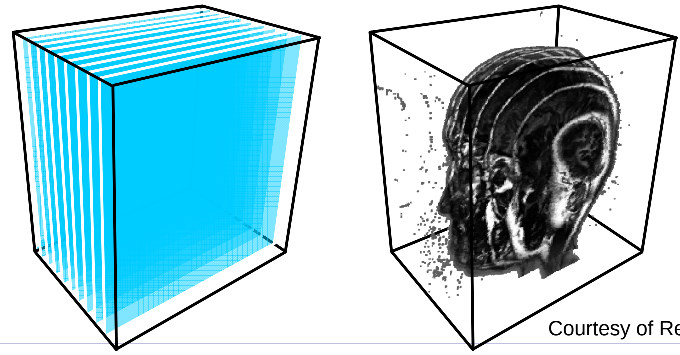
Slice-based method [Rezk-Salama’00]

- represent volume as a stack of 2D slices
- project each slice onto detector plane using texture mapping
- composite buffer to accumulate



Courtesy of Rezk-Salama

Projection: Ping-Pong Buffer

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Buffer = Buffer + Incoming

Avoid simultaneous read and write

Bounce back and forth between two buffers in each pass

```
const GLenum gltex[2] = {GL_COLOR_ATTACHMENT0_EXT, GL_COLOR_ATTACHMENT1_EXT};
int SOURCE_BUFFER = 0;
#define DESTINATION_BUFFER !SOURCE_BUFFER
#define SWAP() SOURCE_BUFFER = DESTINATION_BUFFER
glFramebufferTexture2DEXT(GL_FRAMEBUFFER_EXT, gltex[SOURCE_BUFFER],
   GL_TEXTURE_RECTANGLE_NV, tex[SOURCE_BUFFER], 0);
glFramebufferTexture2DEXT(GL_FRAMEBUFFER_EXT, gltex[DESTINATION_BUFFER],
   GL_TEXTURE_RECTANGLE_NV, tex[DESTINATION_BUFFER], 0);
for (…){
   glDrawBuffer(gltex[DESTINATION_BUFFER);
   glBindTexture(GL_TEXTURE_RECTANGLE_NV, tex[SOURCE_BUFFER]);
   …
   SWAP();
}
```


Projection

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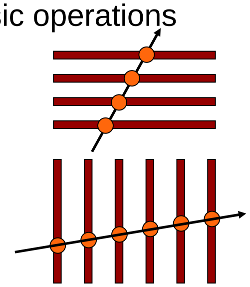
3D transformation & texture mapping are intrinsic operations

Volume slice stacks

- select the slice most parallel to the detector
- non-uniform sampling, on-slice NN/BI
- perform as well as conventional sampling [Xu’06]
  - grid interpolated
  - box line integrated
  - etc.



splat

Projection

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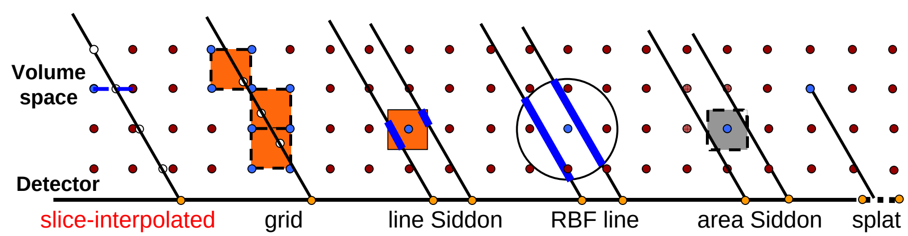
Investigated various schemes in terms of accuracy:



It was shown that the convenient slice-interpolated scheme is qualitatively competitive to the more involved ones listed here.

- see Xu / Mueller, "A comparative study of popular interpolation and integration methods for use in computed tomography," *IEEE 2006 International Symposium on Biomedical Imaging (ISBI '060*

Implementation

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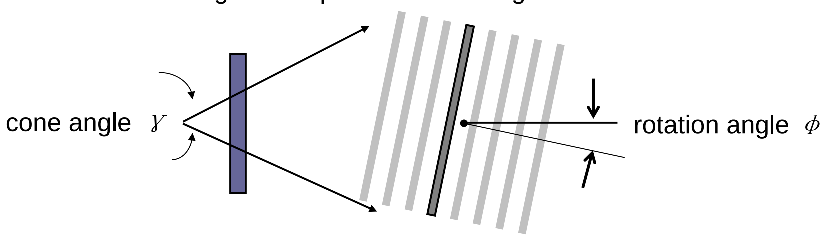
Create polygon proxies for each slice

Associate texture (slice content) with the polygon

Render polygons

- 4 vertices carry voxel coordinates
- 3D transformation is done on GPU, for ALL points in the polygon
- volume slice gets interpolated in the fragment shader



Pixel-Wise Operation

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Correction

- algebraic: scanned projection – simulated projection
- EM: scanned projection / simulated projection

Weighting, slice updating…

```
glMatrixMode(GL_MODELVIEW); glLoadIdentity();
glMatrixMode(GL_PROJECTION); glLoadIdentity();
gluOrtho2D(0, w, 0, h); glViewport(0, 0, w, h);
glBegin(GL_POLYGON);
   glMultiTexCoord2iARB(GL_TEXTURE0_ARB, 0, 0);
   glMultiTexCoord2iARB(GL_TEXTURE0_ARB, w, 0);
   glMultiTexCoord2iARB(GL_TEXTURE0_ARB, w, h);
   glMultiTexCoord2iARB(GL_TEXTURE0_ARB, 0, h);
   … …
glEnd();

float diff   = texRECT(tex0,texcoords0) - texRECT(tex1, texcoords1);
float weight = texRECT(tex2, texcoords2);
float corr   = (weight == 0) ? 0: diff / weight;
```


Attenuation Modeling

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Two sliced volumes

- attenuation A + emission C (under reconstruction)
- first normalize A to [0…1]
- then compute T = 1 - A

Composition

- front-to-back: emission + transparency buffer
- back-to-front: emission buffer (simpler)

Attenuation Modeling: Projection

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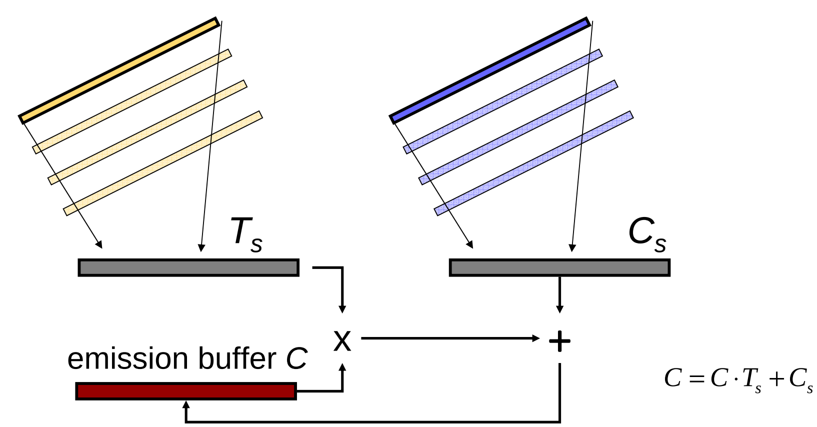
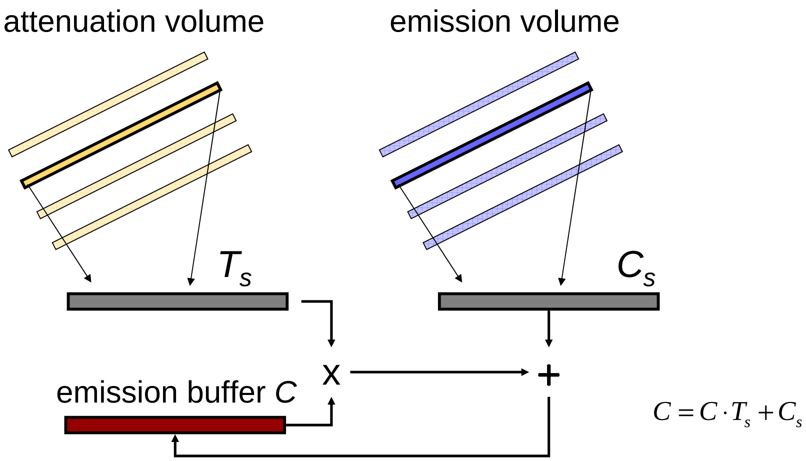
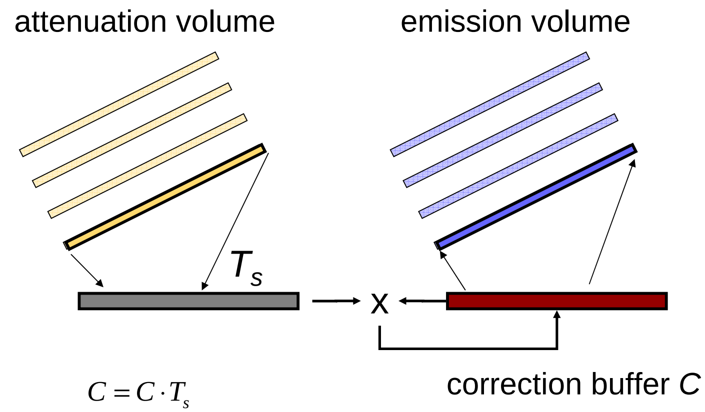


$$C = C \cdot T_s + C_s$$

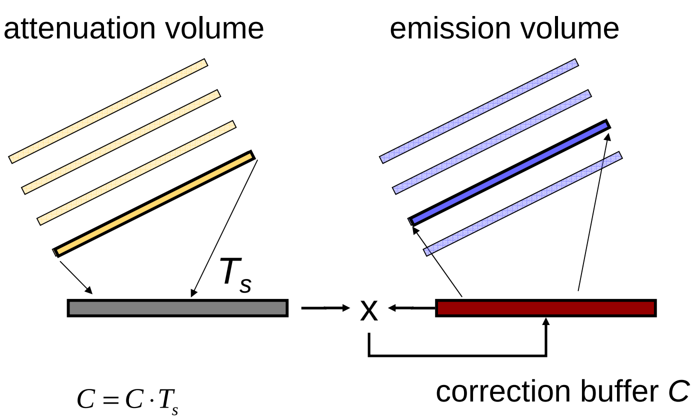
Attenuation Modeling: Projection



Attenuation Modeling: Backprojection



Attenuation Modeling: Backprojection



Scattering Effects

Recursive convolution using a Gaussian filter [Bai'00][Zeng'00]

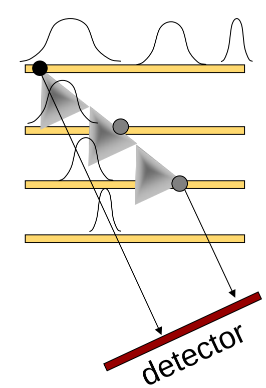
For slice-based projection

- attenuation adjusted kernels
- distance adjusted kernels
- direction adjusted kernels

Projection: Back-to-front order

- adjusted Gaussian blurring
  ➔ scatter energy
- multiply with (1 - attenuation volume)
  ➔ attenuate energy
- add to the slice in the front
  ➔ accumulate energy

Backprojection: Front-to-back



Scattering: Projection

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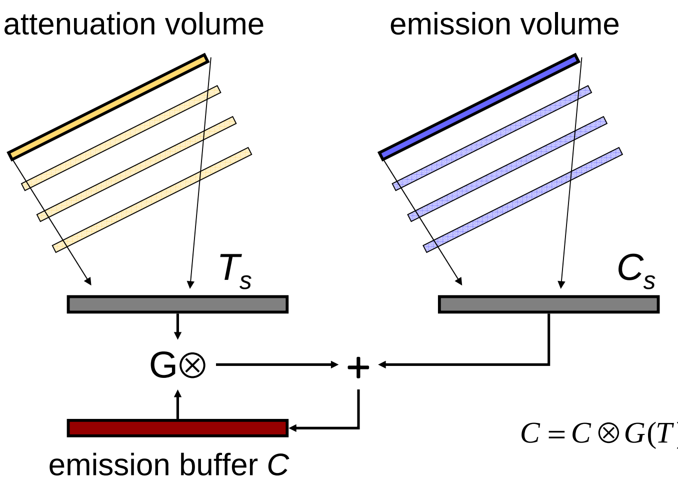

$$C = C \otimes G(T)_s + C_s$$

Scattering: Projection

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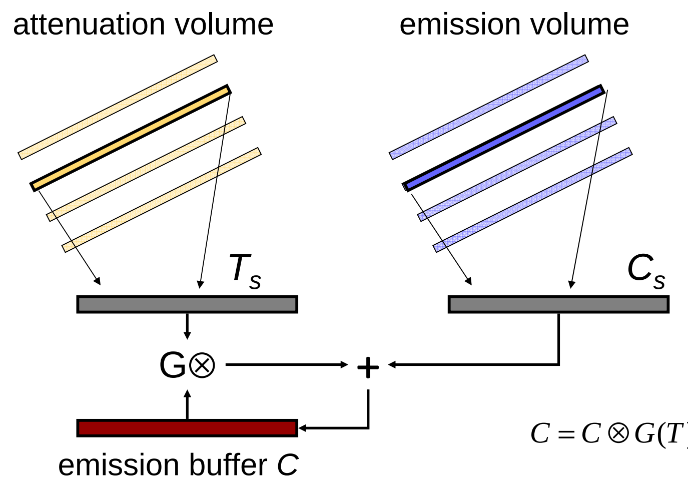

$$C = C \otimes G(T)_s + C_s$$

Scattering: Backprojection

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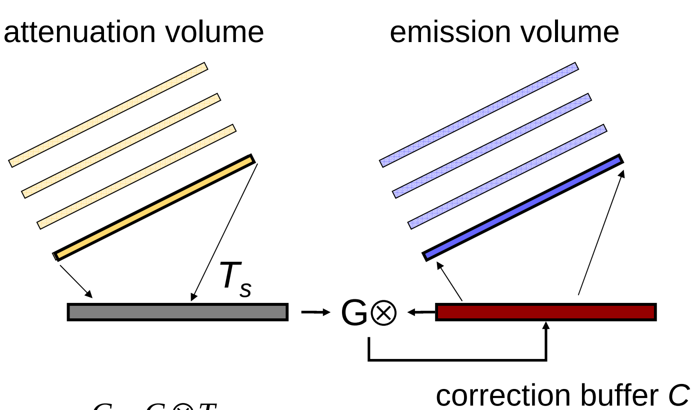

$$C = C \otimes T_s$$

Scattering: Backprojection

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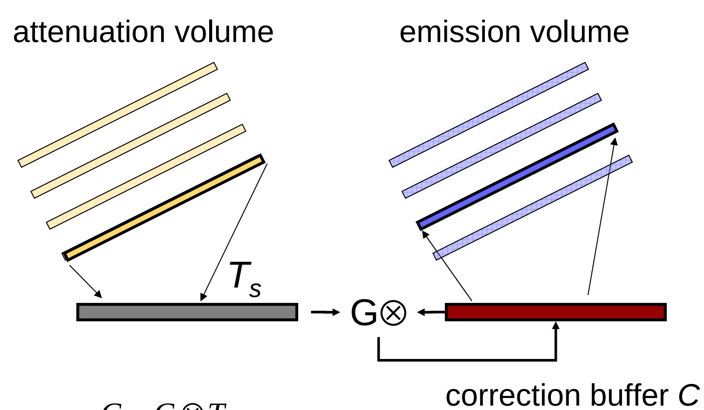

$$C = C \otimes T_s$$

Attenuation + Scattering: Projection

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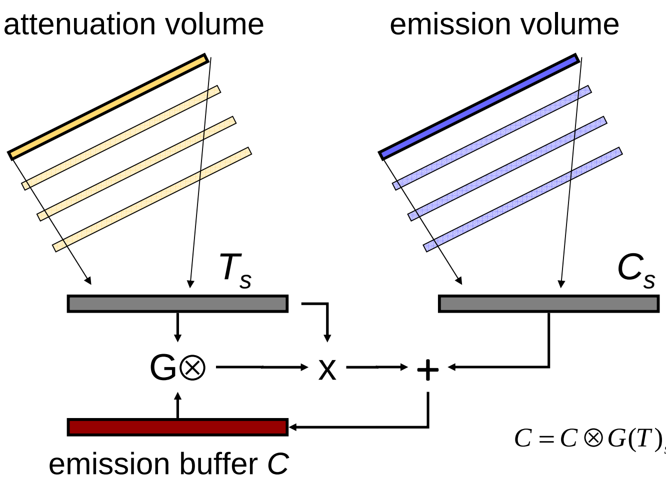

$$C = C \otimes G(T)_s \cdot T_s + C_s$$

Attenuation + Scattering: Projection

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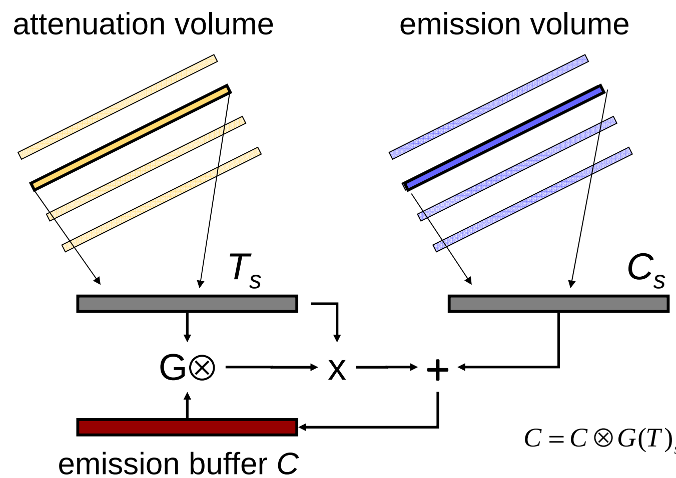
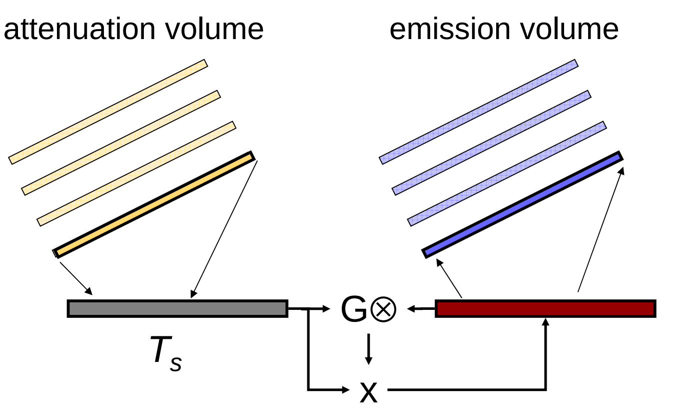


$$C = C \otimes G(T)_s \cdot T_s + C_s$$

Attenuation + Scattering: Backproj.

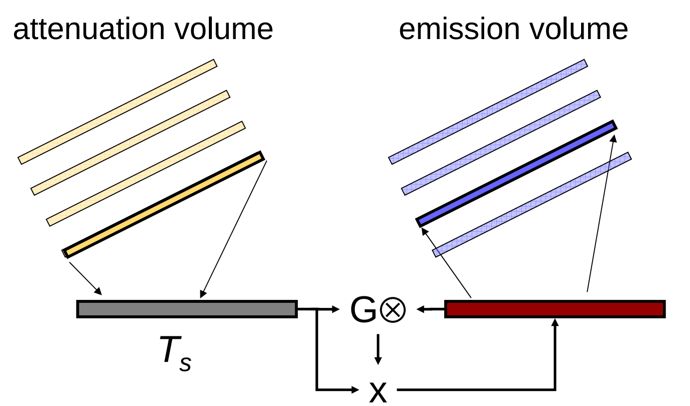
SPIE Medical Imaging



$$C = C \otimes G(T)_s \cdot T_s$$

Attenuation + Scattering: Backproj.

SPIE Medical Imaging



$$C = C \otimes G(T)_s \cdot T_s$$

Texture Operations

We may store transparency and emission volumes in 3D textures

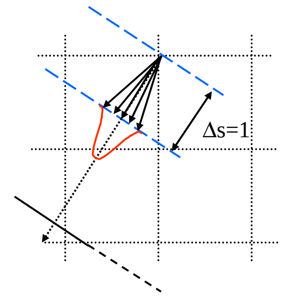
However, 3D textures do not provide efficient write-back capabilities

- solution: perform forward projection with 3D textures and back-projection with 2D textures
- however, 3D/2D texture switching is expensive

Therefore, we use a 2D texture scheme for all operations

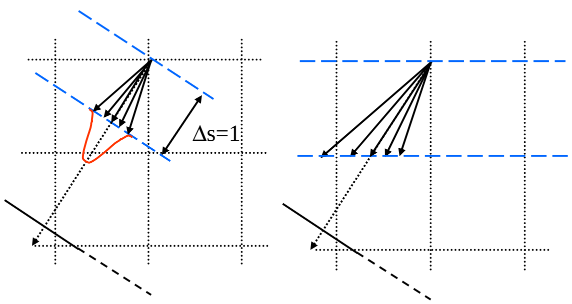
- using an axis-aligned scheme
- but, can this be as accurate than a true 3D texture scheme?

Using 2D Textures



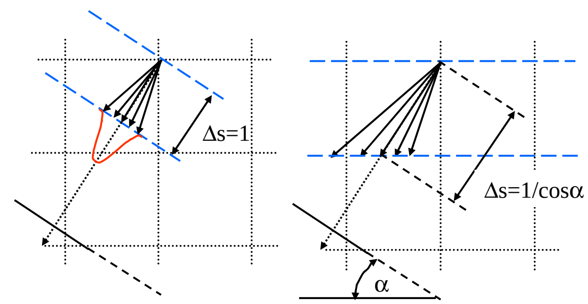
3D texture

Using 2D Textures



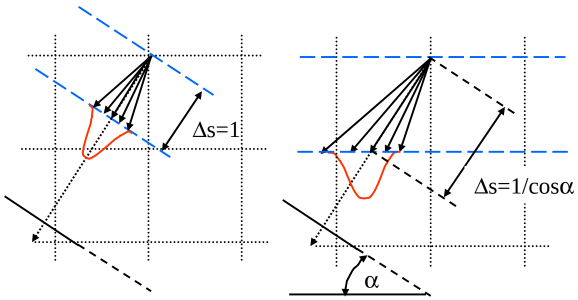
3D texture 2D texture

Using 2D Textures



3D texture 2D texture

Using 2D Textures

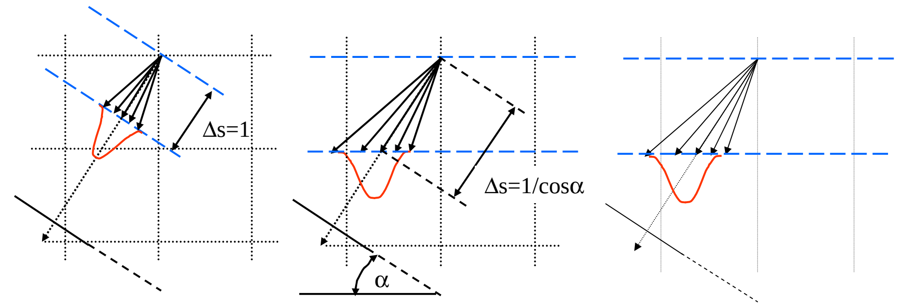


3D texture

2D texture

- scale kernel width by 1/cosα

Using 2D Textures



3D texture

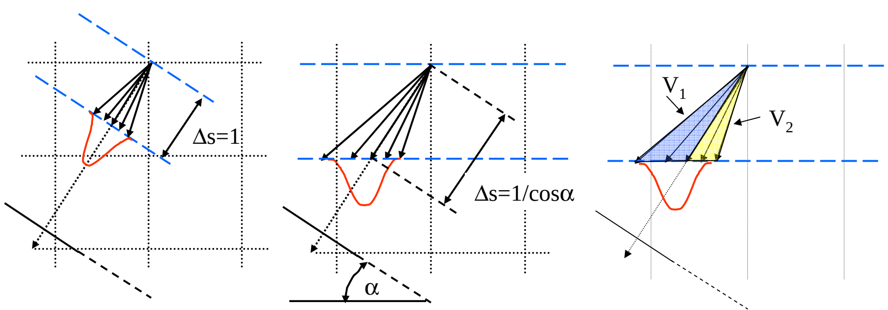
2D texture

- scale kernel width by 1/cosα

2D texture

- scale kernel width by 1/cosα

Using 2D Textures



3D texture

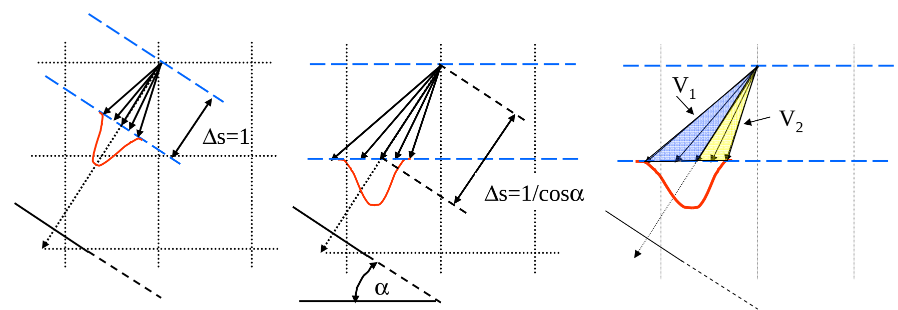
2D texture

- scale kernel width by 1/cosα

2D texture

- scale kernel width by 1/cosα

Using 2D Textures



3D texture

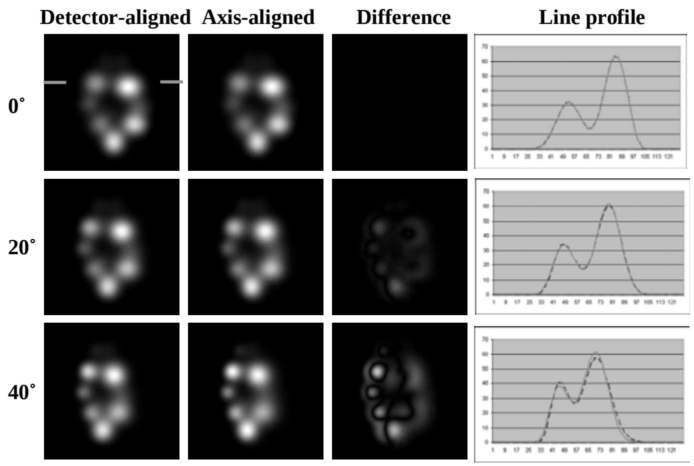
2D texture

- scale kernel width by 1/cosα

2D texture

- scale kernel width by 1/cosα
- adjust kernel width for perspective distortion

Results: Texture Schemes



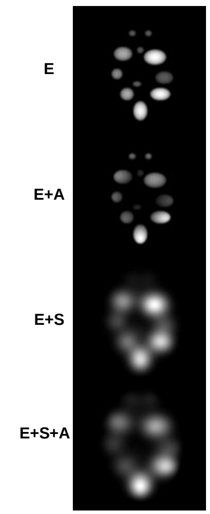
RMS error within 1-2%

Results: Simulations

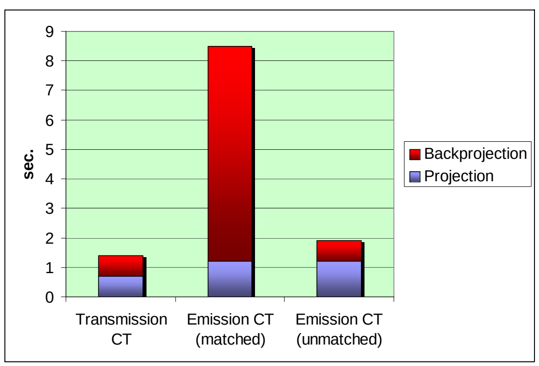
Scattering creates substantially more blur

Attenuation weakens the projections of emissions traversing highly attenuating material

- both with and without scattering.



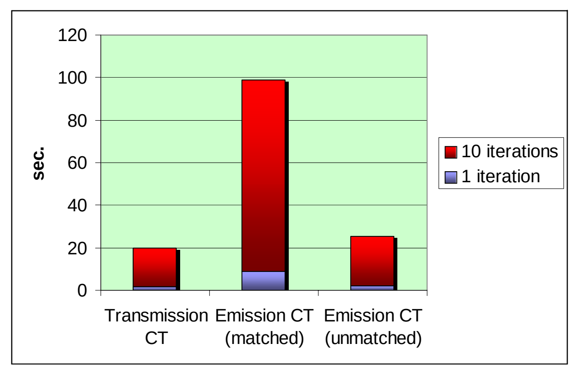
Results: Performance



Adding attenuation and scattering to projector adds little overhead

- compensated back-projector 5 more expensive
- thus, unmatched projector-backprojector significantly more efficient
- will be used for reconstruction

Results: Performance



Adding attenuation and scattering to projector adds little overhead

- compensated back-projector 5 more expensive
- thus, unmatched projector-backprojector significantly more efficient
- will be used for reconstruction

Results: Reconstructions

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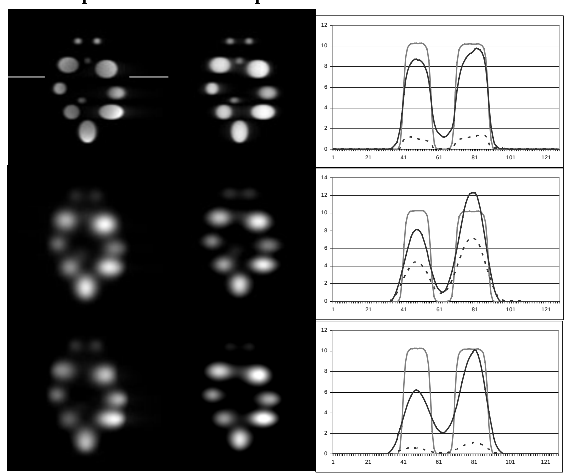

10 iterations

Course Schedule

SPIE Medical Imaging

1:30 – 2:00:	Introduction
2:00 – 2:30:	GPU architecture, programming model, and programming facilties
2:30 – 3:00:	GPU programming examples (image processing)
	Coffee Break
3:30 – 4:00:	CT reconstruction pipeline components
4:00 – 4:30:	GPU-acceleration of individual components
4:30 – 5:00:	Various CT reconstruction pipelines, load balancing and load estimation
5:00 – 5:30:	Reconstruction visualization and final remarks