Network Boundary Detection

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Boundary of a Network

- Boundaries of “holes” in the network
- Outer Boundary
- Inside Vs Outside the network
- Useful in routing, localization, finding the shape of the network...
Boundary of a Network

What is the Definition?
Definition

For a set of points $S$

- **Interior**: $p$ is in the interior if a small disk around $p$ is inside.
- **Exterior**: $p$ is in the exterior if a small disk around $p$ is outside.
- **Boundary**: $p$ is on the boundary if in any disk around $p$, some points are inside, some are outside.
Definition: the difficulty

- Relies on an ambient space: the ‘outside’
- There are no sensors ‘outside’ the network!
- So, who is at the boundary?
Today:


Distance contours break at boundaries

Measure distances from a beacon
Distance contours break at boundaries

Discrete network
Boundary nodes

- Each iso-contour can have several connected components
- Find the end points of each component – boundary nodes
- One beacon is insufficient
Beacon selection

- Pick 1\textsuperscript{st} beacon randomly
- Pick 2\textsuperscript{nd} beacon farthest from first
- Pick 3\textsuperscript{rd} next beacon with max distance to nearer of 1 and 2
- Iterate to pick k beacons

- Number of beacons needed depends on geometry
Application: Landmark Selection in Glider

- Without Boundary knowledge: tiles may contain holes
Landmark on Boundaries
Push Landmarks Away
More Pictures

Low density

UDGs and non UDGs
Boundary Detection with *flowers*

- Try to determine which nodes are *definitely in the interior*

- Nodes that are surrounded on all sides and by sufficiently large number of nodes
Independent Sets and Packing

• Independent nodes: set of nodes, no two of which are adjacent (have a connecting edge)

• If there are $m$ independent nodes in a region, there must be $m$ non-overlapping disks of diameter $= 1$. 
Packing

• Take a cycle of length $k$
• Say $m$ disks of diameter 1 fits inside
• $m$ depends on $k$
• The cycle disconnects inside from outside
  if a connected set has more than $m$ disks, it must be outside
Geometry-Based Reasoning for a Large Sensor Network

Sándor P. Fekete    Alexander Kröller

SwarmNet
Boundary Detection by Topological Methods

- Select a point
- Find the shortest paths to all other points
- Some points have 2 different shortest paths:
  - *Cut locus*
Step 1: Build shortest path tree
Step 2: Find the ‘cut’

- The flow forks near a hole
- Lowest common ancestor – the nearest common point on the path to the root is far
Cut branch

- Set of connected cut nodes
Step 3: Removing cuts

- \#cut branches = \#holes
- Remove cut branches except one: all holes connected together and to exterior – only 1 hole left inside the network
Step 4: Coarse hole boundary

• Find the loop enclosing the hole
  – 2 different path to the cut from the Lowest common ancestor
Step 5: Extremal Nodes

- Distance from the coarse boundary locally maximal
Step 6: Extremal Nodes Must be on Boundary

- Force inner and outer boundaries to go through extremal nodes
Step 7: Restore Cut Nodes
Guarantee

- Provably correct in continuous case
- Approximate by polygons?
Simulations

• Random Distribution of nodes
  – Very good results with avg degree $\geq 10$
  – For very low degrees, take 2-hop/3-hop neighborhoods as ‘fake’ neighbors

• Grid distribution with random perturbation
  – Good results on avg degree $\geq 6$
Random Distributions

Avg deg = 7
1-hop

Avg deg = 7
2-hop (9)

Avg deg = 7
3-hop (12)

Avg deg = 10

Avg deg = 13

Avg deg = 16
Grid with perturbation

Low Density, Sparse Deployment

Avg deg = 6
Avg deg = 8
Avg deg = 12

2628 nodes
Avg deg = 25

1742 nodes
Avg deg = 16

842 nodes
Avg deg = 7
Summary

• Discrete case not formally provable
  – Can we contract a cycle in a network?
• Fast, efficient
• Has been used in network segmentation, localization, many other cases.
• 2-D network: #holes = Topology