Stony Brook University **Midterm 2**
CSE 373 – Analysis of Algorithms **November 22, 2016**

Midterm Exam

Name: _____ Signature: _____

ID #: _____ Circle one: GRAD / UNDERGRAD

INSTRUCTIONS:

- This is a *closed* book, closed mouth exam.
- You must use pencil in the multiple choice component.
- **You must enter the multiple choice on the bubble sheet!!!**
- Check to see that you have 8 pages.
- Write your algorithms in sufficient detail that we can be sure you understand what you are doing.
- Use the back of the pages if you need more room.
- Look over all problems before starting work.
- Think before you write.
- Good luck!!

Problem	Score	Maximum
1		15
2		20
3		15
MC		$34 \times (25/17) = 50$
Total		100

1) (15 points) Describe a backtracking algorithm for efficiently listing all k-element subsets of n items.

For $n = 5$, the 3-element subsets are: (1,2,3), (1,2,4), (1,2,5), (1,3,4), (1,3,5), (1,4,5), (2,3,4), (2,3,5), (2,4,5), (3,4,5).

In particular, first describe the solution vector representation you use, and then describe how you partition the work among construct-candidates, is-a-solution, and process-solution.

2) (20 points) A monotonically non-decreasing subsequence of a sequence S of n numbers is a subsequence T of S such that $T_i \geq T_{i-1}$. Ties are allowed, but not rearranging the order of the elements.

As an example, the longest non-decreasing subsequence of the sequence $S = (2, 5, 2, 8, 7, 3, 1, 6, 4)$ has length 4 and is either (2,2,3,4) or (2,2,3,6).

(a) Give a longest non-decreasing subsequence of $S = \{2, 4, 3, 5, 1, 7, 6, 9, 8\}$. How long is it?

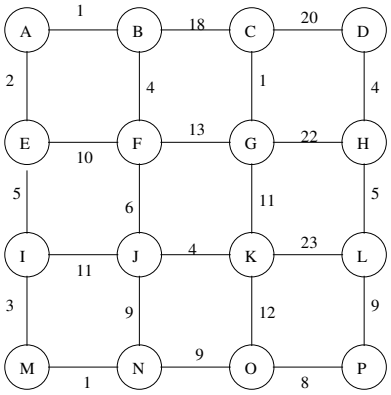
(b) In English, define a function to compute which will permit a dynamic programming algorithm to find the length of the longest non-decreasing subsequence of S.

(c) Give a recurrence relation to compute this function.

(d) What is the running time, if we use dynamic programming to implement the recurrence?

3) (15 points) A minimum *bottleneck* spanning tree of a weighted graph $G = (n, m)$ is a spanning tree which minimizes the length of the longest single edge in the tree.

(a) The graph given below has a minimum bottleneck spanning tree whose longest edge is of length 11. Draw this tree.



(b) Now describe a correct and efficient algorithm to find the minimum bottleneck spanning tree. You must give a **convincing** proof of correctness and give its big Oh running time for full credit.

Multiple Choice

Answer these multiple choice problems in pencil on the bubble sheet! You may write on these pages but they will not be graded.

Mark your test version on the bubble sheet NOW. You have test version **2**.

Neither the TA nor the instructor will answer any questions about these multiple choice problems. Use your best judgement about what they mean.

For all questions on graphs, let n be the number of edges, m be the number of edges, and $d(x)$ be the degree of a given node.

1. What is the running time for generating all the permutations of n distinct numbers using backtracking?

   A. $\Theta(n)$ B. $\Theta(n^2)$ C. $\Theta(2^n)$ D. $\Theta(n!)$

2. Which of these can be a possible output order of a backtracking algorithm to construct permutations of $\{1, 2, 3\}$?

   A. 2,3,1; 2,1,3; 1,2,3; 3,2,1; 3,1,2; 1,3,2
   B. 3,1,2; 3,2,1; 1,3,2; 1,2,3; 2,1,3; 2,3,1
   C. 2,1,3; 2,3,1; 3,2,1; 1,3,2; 3,1,2; 1,2,3
   D. 1,2,3; 2,3,1; 1,3,2; 2,1,3; 3,1,2; 3,2,1

3. Let $a_1, a_2, ..., a_n$ be a sequence of numbers. An increasing subsequence is a subsequence of not-necesssarily adjacent numbers which are getting strictly larger. What is the length of the longest increasing subsequence of $\{5, 2, 8, 6, 3, 9, 4, 1, 7\}$?

   A. 3 B. 4 C. 5 D. 6

4. What is the edit distance of the following two strings?
   string 1: SNOWY
   string 2: SUNY

   A. 2 B. 3 C. 4 D. 5

5. Which is the fastest algorithm for finding the shortest path between two vertices in an unweighted graph?

   A. Breadth-first search
   B. Depth-first search
   C. Dijkstra's shortest path
   D. Floyd's algorithm

6. Which of the following is true of every DAG with n vertices?

   A. every vertex is an articulation vertex
   B. there are $o(n^2)$ edges
   C. there are $\Omega(n)$ edges
   D. there is at least one topological ordering of the vertices

7. Which of the following is true of every tree with n vertices?

   A. every vertex is an articulation vertex
   B. the adjacency list representation takes $\mathcal{O}(n)$ space.
   C. it is strongly-connected

D. there is at most one topological ordering of the vertices

8. What is the running time to compute the following recurrence, if properly implemented with dynamic programming, as a function of n?

$$F(n) = \sum_{i=1}^{n-1} (F(i) \times F(i-1))$$

A. $\Theta(n)$ B. $\Theta(n \log n)$ C. $\Theta(n^2)$ D. $\Theta(n^3)$

9. What is the running time to compute the following recurrence, if properly implemented with dynamic programming, as a function of n and k?

$$F(n,k) = \sum_{i=0}^{n-1} \sum_{j=0}^{k} F(i,j)$$

A. $\Theta(nk)$ B. $\Theta(n^2k)$ C. $\Theta(n^2k^2)$ D. $\Theta(n^3k^3)$

10. Recall that Prim's algorithm keeps track of the distance from each node not yet in the minimum spanning tree (MST) to its closest neighbor in the MST, while Kruskal's algorithm sorts the set of edges and repeated adds nodes using union find to determine if they are already in the same component.

Kruskal's algorithm performs better on _____ graphs, while Prim's performs better on _____ ones.

A. dense, sparse B. sparse, dense C. cyclic, acyclic D. acyclic, cyclic

11. What is the *space* complexity of Djikstra's algorithm, not counting the input graph?

A. $O(n)$ B. $O(n \log n)$ C. $O(1)$ D. $O(m)$

12. The stack data structure is largely associated with what graph traversal algorithm?

A. Topological Sort B. Breadth-First Search C. Depth-First Search D. Minimum Spanning Tree

13. The queue data structure is largely associated with what graph traversal algorithm?

A. Topological Sort B. Breadth-First Search C. Depth-First Search D. Minimum Spanning Tree

14. A graph with a cycle is traversed to produce a search tree. All of the non-tree edges in the cycle connect to ancestors. What graph traversal algorithm was used to traverse the graph?

A. Topological Sort B. Breadth-First Search C. Depth-First Search D. Minimum Spanning Tree

15. An undirected graph is traversed to produce a search tree. All of the non-tree edges in each cycle connect to siblings on the same level. What graph traversal algorithm was used to traverse the graph?

A. Topological Sort B. Breadth-First Search C. Depth-First Search D. Minimum Spanning Tree

16. Students must take certain prerequisite classes before taking advanced courses. What type of algorithm can be run to provide a valid order of classes to take?

A. Topological Sort B. Breadth-First Search C. Depth-First Search D. Minimum Spanning Tree

17. In an unweighted graph, which of the following algorithms does *not* produce a minimum spanning tree?

    A. Topological Sort B. Breadth-First Search C. Kruskal's Algorithm D. Prim's Algorithm

18. Which algorithm *does not* allow for negative edge weights when computing an answer?

    A. Kruskal's algorithm B. Dijkstra's algorithm C. Prim's algorithm D. Connected components

19. When constructing all permutations of n elements, what is the advantage of Backtrack-DFS over a simple BFS?

    A. Space complexity B. Time complexity C. Simplicity of programming D. No real advantage

20. One issue with developing an algorithm for games with large search trees, such as Chess, is the exponential number of possibilities for future moves. What technique can be used to remove some possibilities when searching?

    A. Pruning B. Simulated Annealing C. Parallel Algorithms D. Backtracking

21. What is the best achievable space complexity for an $O(n)$ algorithm that computes the Fibonnaci numbers using dynamic programming?

    A. $O(1)$ B. $O(n)$ C. $O(n^2)$ D. $O(2^n)$

22. You are tasked with finding the maximum number in a set of values, A, using dynamic programming. Which recurrence gives a valid definition of $Max(x)$?

    A. $Max(x) = max(A[x], A[x-1])$
    B. $Max(x) = max(A[x-1], A[x-2])$
    C. $Max(x) = max(A[x-1], Max(x))$
    D. $Max(x) = max(A[x], Max(x-1))$

23. If the pre-order traversal sequence of a tree T is $ACDEFHGB$, the in-order traversal sequence is $DECAHFBG$, then what is the sequence of post-order traversal?

    A. $HGFEDCBA$ B. $EDCHBGFA$ C. $BGFHEDCA$ D. $EDCBGHFA$

24. A simple undirected graph of 8 vertices can have at most ( ) edges.

    A. 14 B. 28 C. 56 D. 112

25. A connected, undirected graph of 8 vertices must have at least ( ) edges.

    A. 5 B. 6 C. 7 D. 8

26. The time complexity of the Floyd-Warshall algorithm is ( ).

    A. $O(n)$ B. $O(n \log n)$ C. $O(n^2)$ D. $O(n^3)$

27. Backtracking is essentially ( ).

    A. a recursive algorithm B. an iterative algorithm C. BFS D. DFS

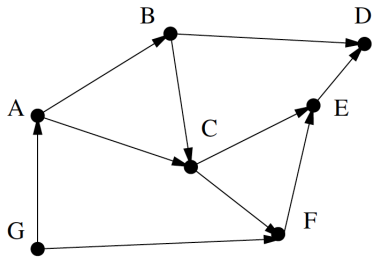


Figure 1: Graph 1.

28. For the graph shown in Figure 1, suppose vertices A, B, C, D, E, F, G are stored in alphabetical order in this graph. What is DFS order of traversing this graph starting from vertex G?

    A. G, A, B, C, D, E, F B. G, A, B, D, C, E, F C. G, A, B, C, E, D, F D. G, A, B, C, F, E, D

29. For the graph shown in Figure 1, suppose vertices A, B, C, D, E, F, G are stored in alphabetical order in this graph. What is BFS order of traversing this graph starting from vertex G?

    A. G, A, F, B, C, D, E B. G, A, B, C, F, D, E C. G, A, C, F, B, E, D D. G, A, F, B, C, E, D

30. Which of the following does *not* involve DFS traversal on a graph?

    A. Finding cycles. B. Finding articulation vertices. C. Find the shortest path from the root to a node. D. Topological sorting of vertices.

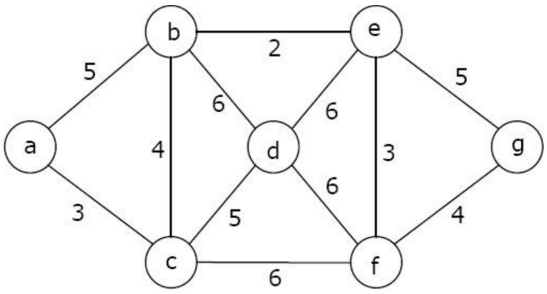


Figure 2: Graph 2.

31. When running Prim's algorithm on Graph 2 to find the minimum spanning tree, what can be the vertex order of this algorithm staring from vertex a?

    A. a, c, b, d, e, f, g B. a, c, b, e, d, f, g C. a, c, d, e, b, f, g D. a, c, b, e, f, g, d

32. What is the total weight of the minimum spanning tree of Graph 2?

    A. 21 B. 22 C. 23 D. 24

33. When running Dijkstra's algorithm on Graph 2 to find the shortest path from a to g, what can be the vertex order of this algorithm staring from vertex a?

    A. a, c, b, d, e, f, g B. a, c, b, e, d, f, g C. a, c, d, e, b, f, g D. a, c, b, e, f, g, d

34. What is the length of the shortest path from a to g in Graph 2?

    A. 3 B. 11 C. 12 D. 14