

CSE537

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# AIMA CHAPTER 10.3: PLANNING GRAPHS

Resource: based on material & slide  
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# SEARCH AND PLANNING

- × **Planning:** generate seq. of actions to achieve one's goals
- × We have seen two examples of planning agents so far:
  - + search-based problem-solving agent of Ch.3
    - × can find sequences of actions that result in a goal state.
    - × but deals with atomic states (needs good domain-specific heuristics)
  - + hybrid logical agent of Chapter 7.
    - × can find plans without domain-specific heuristics
      - (uses domain-independent heuristics based on the logical structure of the problem)
    - × but relies on ground (variable-free) propositional inference
      - (it may be over worked when there are many actions and states.)
- × We want representation for **planning problems**
  - + that **scales up** to problems unable to be handled by earlier approaches.

# CLASSICAL PLANNING ENVIRONMENT

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The assumptions for classical planning problems

- × Fully observable
  - + we see everything that matters
- × Deterministic
  - + the effects of actions are known exactly
- × Static
  - + no changes to environment other than those caused by agent actions
- × Discrete
  - + changes in time and space occur in quantum amounts
- × Single agent
  - + no competition or cooperation to account for

# FACTORED REPRESENTATION IN PLANNING LANGUAGE

- × What is a good representation?
  - + Expressive enough to describe a wide variety of problems
  - + Restrictive enough for efficient algorithms to operate on it
  - + Planning algorithm should be able to take advantage
    - × of the *logical structure* of the problem
- × Historical AI planning languages
  - + STRIPS was used in classical planners
    - × Stanford Research Institute Problem Solver
  - + ADL addresses expressive limitations of STRIPS
    - × Action Description Language
    - × Adds features not in STRIPS
      - ★ negative literals, quantified variables, conditional effects, equality
  - + We'll look at a simpler version of de facto standard language called PDDL

# PDDL

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- × PDDL and most of the planning language use *factored* representation for states
  - + Each state is represented as a collection of variables
- × Planning Domain Definition Language
  - + To see its expressive power, recall propositional agent in the Wumpus World, which requires  $4Tn^2$  actions to describe a movement of 1 square
  - + PDDL captures this with a single Action Schema

# PDDL: STATE

- × Each **state** is represented as a conjunction of fluents: ground, functionless atoms.
  - + Ex>  $Poor \wedge Unknown$  might represent the state of a hapless agent,
  - + Ex> a state in a package delivery problem might be  $At(Truck1, Melbourne) \wedge At(Truck2, Sydney)$
- × **Database semantics** is used
  - + the **closed-world assumption**: any fluents that are not mentioned are false,
  - + the **unique names assumption**: ex> $Truck1$  and  $Truck2$  are distinct
  - + fluents *not* allowed:  $At(x, y)$  (because it is non-ground),  $\neg Poor$  (because it is a negation), and  $At(Father(Fred), Sydney)$  (because it uses a function symbol).
- × This state representation allows alternative algorithms
  - + it can be manipulated either by *logical inference* techniques or by
  - + *set operations* (sets may be easier to deal with)

# PDDL: ACTION SCHEMAS

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- × **Actions** are defined by a set of *action schemas*
  - + These implicitly define the ACTIONS(s) & RESULT(s, a) functions required to apply search techniques
- × Classical planning concentrates on problems where most actions leave most things unchanged.
  - + PDDL specify the result of an action in terms of what changes;  
everything that stays the same is left unmentioned.

# PDDL: ACTION SCHEMAS

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- × Ground (variable-free) actions are represented by single **action schema** - a *lifted* representation
  - + lifts from propositional logic to a restricted subset of First-order logic
- × Consists of
  - + the schema name,
  - + list of variables used,
    - × Consider variables as universally quantified, choose any values we want to instantiate them
  - + a **precondition**
    - × PRECOND: defines states in which an action can be executed
  - + an effect
    - × EFFECT: defines the result of executing the action



# EXAMPLE ACTION SCHEMA

- × Each represents a set of variable-free actions
  - + Form: Action Schema = predicate + preconditions + effects
  - + Example action schema for flying a plane from one location to another :

*Action(Fly(p, from, to),*

*PRECOND: At(p, from)  $\wedge$  Plane(p)  $\wedge$  Airport(from)  $\wedge$  Airport(to)*

*EFFECT:  $\neg$ At(p, from)  $\wedge$  At(p, to)*

- + Action that results from substituting values for all the variables:

*Action(Fly(P1,SFO,JFK),*

*PRECOND:At(P1,SFO)  $\wedge$  Plane(P1)  $\wedge$  Airport(SFO)  $\wedge$  Airport(JFK)*

*EFFECT: $\neg$ At(P1,SFO)  $\wedge$  At(P1,JFK)*

# APPLYING ACTION SCHEMA

- × Action  $a$  is *applicable* in state  $s$ 
  - +  $s$  entails the precondition of  $a$ 
    - × If  $a$ 's preconditions are satisfied in  $s$  (" $a$  is *applicable* in  $s$ ")  
 $a \in \text{ACTIONS}(s) \Leftrightarrow s \models \text{PRECOND}(a)$
  - + Given variables in  $a$ , there can be multiple applicable instantiations
    - × For  $v$  variables in a domain with  $k$  unique object names, worst case time to find applicable ground actions is  $O(v^k)$
- + Leads to one approach for solving PDDL planning problems
  - × **Propositionalize** by replacing action schemas with sets of ground actions
    - then applying a propositional solver like SATPlan
  - × Impractical for large  $v$  &  $k$

# PDDL: RESULT

- × Result of executing action  $a$  in state  $s$  is state  $s'$   
$$RESULT(s, a) = (s - DEL(a)) \cup ADD(a)$$
  - + Start with  $s$
  - + Remove negative literal in the action's effect  
(the *delete list*,  $DEL(a)$ )
  - + Add positive literals in action's EFFECTS  
(the *add list*,  $ADD(a)$ )
  - + For example, with the action  $Fly(P1, SFO, JFK)$ ,
    - × we would remove  $At(P1, SFO)$  and
    - × add  $At(P1, JFK)$ .
- × Any variable in the effect must also appear in the precondition.
  - + When the precondition is matched against the state  $s$ , all the variables will be bound, and  $RESULT(s, a)$  will therefore have only ground atoms.

# PDDL: ACTION SCHEMAS

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## 1. Variables & ground terms

- + Variables in effects must also be in precondition
  - × so matching to state  $s$  yields results with all variables bound i.e. that contain only ground terms
  - × Ground states are closed under the RESULT operation.

## 2. Handling of time

- + No explicit time terms
- + Instead time is implicitly represented in PDDL schemas
  - × Preconditions always refer to time:  $t$
  - × Effects always refer to time:  $t + 1$

## 3. A set of schemas defines a *planning domain*

- + A specific *problem* within the domain is defined with the addition of an initial state and a goal.

# PDDL: INITIAL STATES, GOALS, SOLUTIONS

- × Initial state
  - + Conjunction of ground terms
- × Goal
  - + Conjunction of positive and negative literals that contain variable.
    - × Both ground terms & those containing variables
    - ×  $\exists p. \text{At}(p, \text{SFO}) \wedge \text{Plane}(p)$ .
  - + Variables are treated as existentially quantified
    - ×  $\exists p$  so this goal is to have *any* plane at SFO
- × Solution
  - + A sequence of actions ending in  $s$  that entails the goal
  - +  $\exists s$  state  $\text{Rich} \wedge \text{Famous} \wedge \text{Miserable}$  entails the goal  $\text{Rich} \wedge \text{Famous}$ ,
  - +  $\exists p$  state  $\text{Plane}(P1) \wedge \text{At}(P1, \text{SFO})$  entails  $\text{At}(p, \text{SFO}) \wedge \text{Plane}(p)$
- × We have defined planning as a search problem:
  - + have an initial state, an ACTIONS function, a RESULT function, and a goal test



# WHY PLANNING GRAPHS

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- × All of the heuristics we have suggested can suffer from inaccuracies.
- × A special data structure called a **planning graph** can be used to give better heuristic estimates.
- × We can search for a solution over the space formed by the planning graph, using an algorithm called **GRAPH PLAN**.
  - + These heuristics can be applied to any of the search techniques we have seen so far.

# PLANNING GRAPHS

- × Graphplan was developed in 1995 by Avrim Blum and Merrick Furst, at CMU.
- × Constructs compact **constraint encoding** of state space from operators and initial state, which prunes many invalid plans.
- × A planning graph compactly encodes the space of consistent plans, while **pruning** . . .
  - + Partial states and actions at each time  $i$  that are **not reachable** from the **initial state**.
  - + Pairs of actions and propositions that are **mutually inconsistent** at time  $i$ .
  - + Plans that **cannot reach the goals**.



# PLANNING GRAPHS PROPERTIES

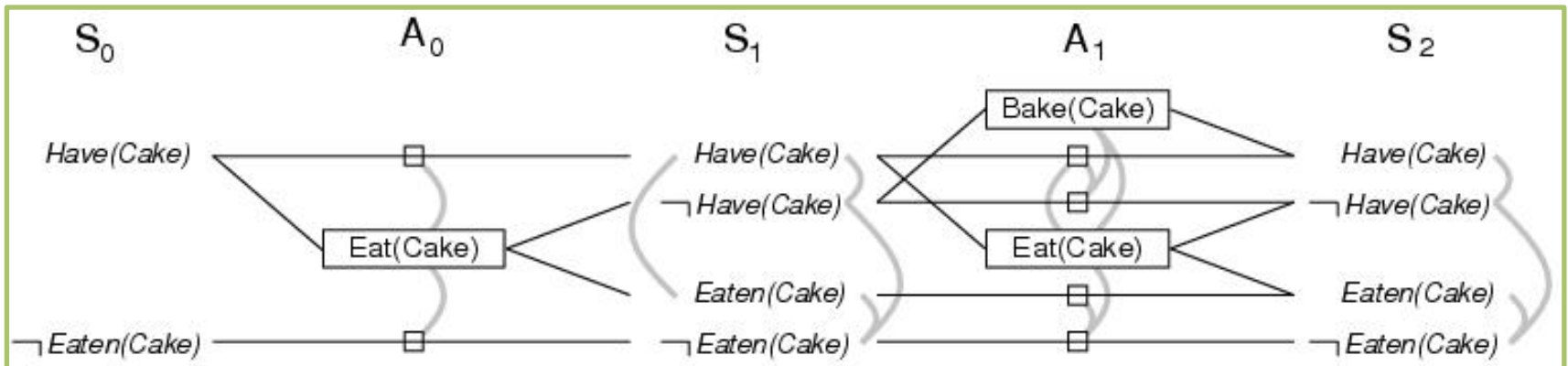
- × A **polynomial-size approximation** to tree-based state space searching that can be constructed quickly
- × The plan graph does not eliminate all infeasible plans.
- × Planning graph cannot answer definitely whether goal  $G$  is reachable from initial state  $S_0$ , but it can estimate how many steps it takes to reach the goal.
  - + Always correct when it reports the goal is not reachable
  - + Never overestimate the number of steps (**admissible heuristic**)
- × Planning graphs
  - + Provide a possible basis for better search heuristics
  - + Can be use directly, for extracting a solution to a planning problem, by applying the **GRAPHPLAN algorithm**

# Problem “Have cake and eat cake too”

## PDDL Problem Description

```
Init(Have(Cake))
Goal(Have(Cake)  $\wedge$  Eaten(Cake))
Action(Eat(Cake))
  PRECOND: Have(Cake)
  EFFECT:  $\neg$ Have(Cake)  $\wedge$  Eaten(Cake))
Action(Bake(Cake))
  PRECOND:  $\neg$  Have(Cake)
  EFFECT: Have(Cake))
```

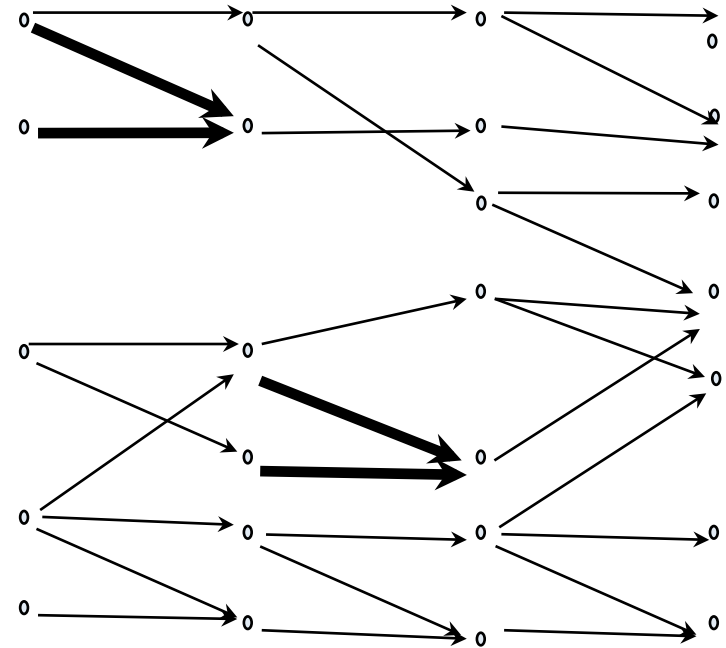
## corresponding planning graph



# PLANNING GRAPH DESCRIPTION

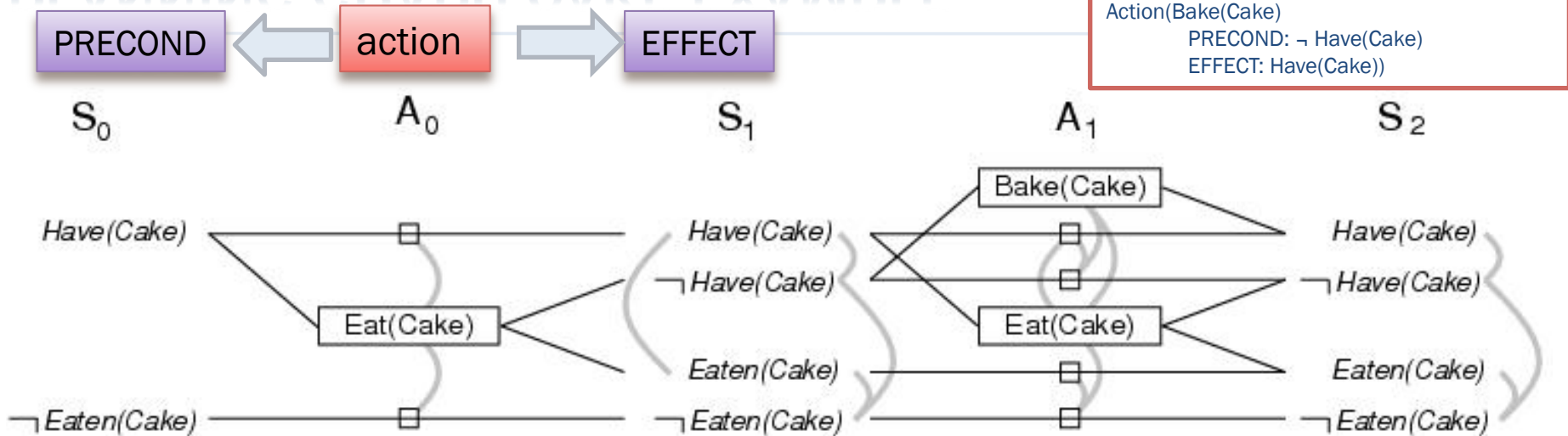
## × Planning graph

- + Is a directed graph organized in time steps **levels**
- + Consist of alternating
  - ×  **$S_i$  level**: contains all the literals that could result from any possible choice of action in  $A_{i-1}$
  - ×  **$A_i$  level**: contains all the actions that are applicable in  $S_i$ .
  - × **Precondition link**
  - × **Effects link**
  - × **Mutual exclusion (mutex) link**  
s: links joining nodes that can not persist simultaneously



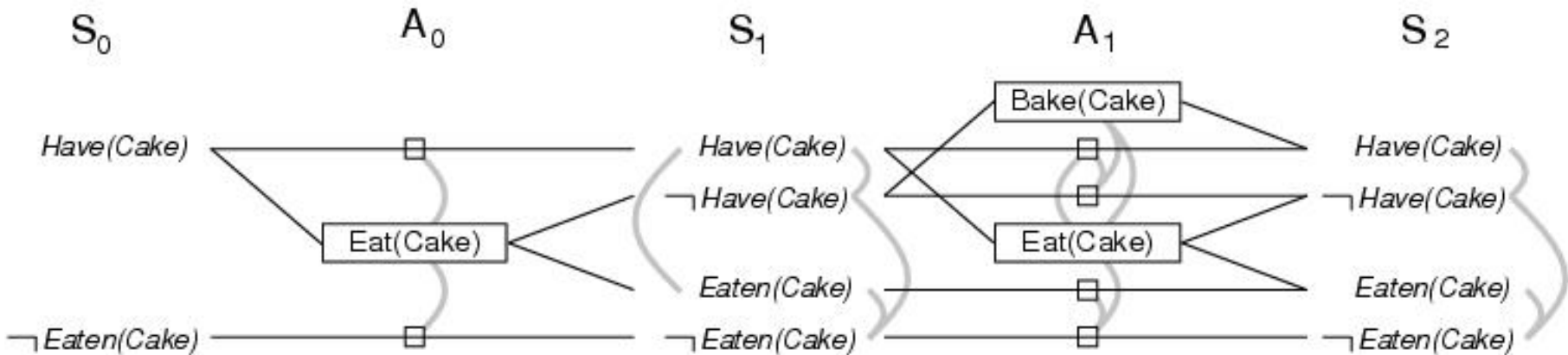
Proposition	Action	Proposition	Action
Init State	Time 1	Time 1	Time 2

# PLANNING GRAPH CAKE EXAMPLE



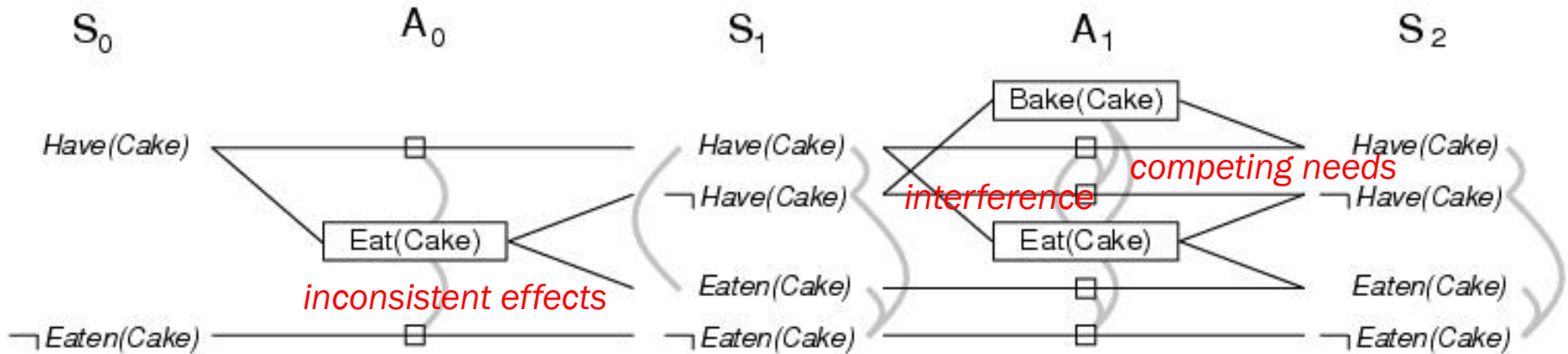
- Start at level  $S_0$ , determine action level  $A_0$  & next level  $S_1$ 
  - $A_0$ : all actions whose preconditions are satisfied in the previous level (i initial state)
    - Lines connect PRECONDs at  $S_0$  to EFFECTs at  $S_1$
  - Also, for each literal in  $S_i$ , there's a *persistence action* (square box) & line to it in the next level  $S_{i+1}$
- Level  $A_0$  contains the actions that *could* occur
  - Conflicts between actions are represented by arcs: **mutual exclusion or mutex links**

# PLANNING GRAPH CAKE EXAMPLE



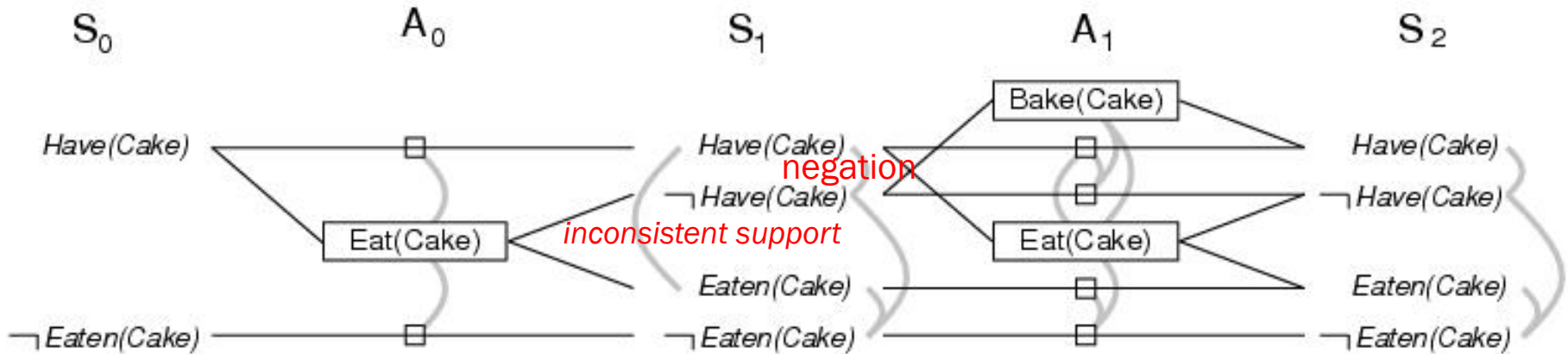
- Level  $S_1$  contains all the literals that could result
  - From picking any subset of actions in  $A_0$
  - So  $S_1$  is a **belief state** consisting of the set of all possible states
    - Each is a **subset of literals with no mutex links between members**
  - Conflicts between literals that cannot occur together are represented by the mutex links.
- The level generation process is repeated
- Termination condition (**leveling off**):
  - When consecutive levels are identical

# MUTEX LINKS – ACTION



- Mutex relation holds between 2 actions at a level when
  - 1. **Inconsistent effects**
    - One action negates the effect of another
    - $\text{Eat}(\text{Cake})$  and  $\text{Have}(\text{Cake})$  have inconsistent effects because they disagree on the effect  $\text{Have}(\text{Cake})$ .
  - 2. **Interference**
    - An effect of one action negates a precondition of the other;
    - Ex>  $\text{Eat}(\text{Cake})$  interferes with the persistence of  $\text{Have}(\text{Cake})$  by negating its precondition.
  - 3. **Competing needs**
    - A precondition of one action is mutex with a precondition of the other
    - Ex>  $\text{Bake}(\text{cake})$  &  $\text{Eat}(\text{cate})$  <- compete on the value of  $\text{Have}(\text{cake})$

# MUTEX LINKS - LITERALS



- Mutex relation holds between 2 literals at a level when
  - 1. One is the **negation** of the other
  - 2. **Inconsistent support**
    - If each possible action pair that could achieve the literals is mutex
    - Ex>  $Have(Cake)$  &  $Eaten(Cake)$  at  $S_1$
    - (the only way of achieving  $Have(Cake)$ , the persistence action, is mutex with the only way of achieving  $Eaten(Cake)$ , )

# PLANNING GRAPHS COMPLEXITY

- × Construction has complexity polynomial in the size of the planning problem:

$$O(n(a + l)^2)$$

- × Given  $l$  literals and  $a$  actions,
  - × each  $S_i$  has no more than
    - \*  $l$  nodes and
    - \*  $l^2$  mutex links, and
  - × each  $A_i$  has no more than
    - \*  $a + l$  nodes (including the no-ops),
    - \*  $(a + l)^2$  mutex links, and
    - \*  $2(al + l)$  precondition and effect links.
- × entire graph with  $n$  levels has a size of  $O(n(a + l)^2)$



# PROPERTIES OF COMPLETED PLANNING GRAPH

- × Provides information about the problem & candidate heuristics
- × A goal literal  $g$  that does not appear in the final level cannot be achieved by any plan
- × The **level cost**, the level at which a goal literal first appears, is useful as a cost estimate of achieving that goal literal
- × Note that level cost is **admissible**, though possibly **inaccurate** since it counts levels, not actions
  - + Planning graphs allow several actions at each level, whereas the heuristic counts just the level and not the number of actions.
  - + We could find a better alternative level cost by using a **serial planning graph** variation, restricted to one action per level
    - × Add mutex links between every pair of nonpersistence actions

# PLANNING GRAPHS & HEURISTICS

- × Planning Graph provides
  - + Possible heuristics for the cost of a *conjunction of goals*
  - + 1. *Max-level* heuristic : highest level of any conjunct in the goal
    - × Admissible, possibly not accurate
  - + 2. *Level sum* heuristic: the sum of level costs of conjuncts in the goal
    - × Incorporates the subgoal independence assumption
      - \* So may be inadmissible to degree the assumption does not hold
      - \* Works well in practice for problems that are largely decomposable
  - + 3. *Set-level* heuristic: level where all goal conjuncts are present without mutex links
    - × Admissible,
    - × Dominates the max-level heuristic
    - × Works well on tasks with good deal of interaction among subplans.
    - × However, ignores interactions among three or more literals.

# PLANNING GRAPHS & HEURISTICS

- × A Planning Graph is a relaxed version of the problem
  - + If a goal literal  $g$  does not appear, no plan can achieve it,
  - + If it does appear, is not guaranteed to be achievable
  - + Why?
    - × The PG only captures pairwise conflicts & there could be higher order conflicts likely not worth the computational expense of checking for them
      - ★ Similar to Constraint Satisfaction Problems where arc consistency was a valuable pruning tool
    - × 3-consistency or even higher order consistency would have made finding solutions easier but was not worth the additional work
  - + Example where PG fails to detect unsolvable problem
    - × Blocks world problem with goal of A on B, B on C, C on A
      - ★ Any pair of subgoals are achievable, so no mutexes
      - ★ Problem only fails at stage of searching the PG

# THE GRAPHPLAN ALGORITHM

## × GRAPHPLAN algorithm

+ Generates the Planning Graph & extracts a solution directly

```

function GRAPHPLAN(problem) return solution or failure
  graph ← INITIAL-PLANNING-GRAPH(problem)
  goals ← CONJUNCTS(problem. GOAL)
  nogoods ← an empty hash table
  for tl = 0 to ∞ do
    if goals all non-mutex in  $S_t$  of graph then
      solution ← EXTRACT-SOLUTION(graph, goals, NUMLEVELS(graph), nogoods)
      if solution ≠ failure then return solution
    if graph and nogoods have both leveled off then return failure
  graph ← EXPAND-GRAPH(graph, problem)
  
```

EXTRACT-SOLUTION: search for a plan that solves the problem.

EXPAND-GRAPH: adds a new level

# EXAMPLE: SPARE TIRE PROBLEM

## × PDDL of spare tire problem (problem of changing a flat tire)

Init( $\text{At}(\text{Flat}, \text{Axle}) \wedge \text{At}(\text{Spare}, \text{Trunk})$ )

Goal( $\text{At}(\text{Spare}, \text{Axle})$ )

Action( $\text{Remove}(\text{Spare}, \text{Trunk})$ )

PRECOND:  $\text{At}(\text{Spare}, \text{Trunk})$

EFFECT:  $\neg \text{At}(\text{Spare}, \text{Trunk}) \wedge \text{At}(\text{Spare}, \text{Ground})$ )

Action( $\text{Remove}(\text{Flat}, \text{Axle})$ )

PRECOND:  $\text{At}(\text{Flat}, \text{Axle})$

EFFECT:  $\neg \text{At}(\text{Flat}, \text{Axle}) \wedge \text{At}(\text{Flat}, \text{Ground})$ )

Action( $\text{PutOn}(\text{Spare}, \text{Axle})$ )

PRECOND:  $\text{At}(\text{Spare}, \text{Ground}) \wedge \neg \text{At}(\text{Flat}, \text{Axle})$

EFFECT:  $\text{At}(\text{Spare}, \text{Axle}) \wedge \neg \text{At}(\text{Spare}, \text{Ground})$ )

Action( $\text{LeaveOvernight}$ )

PRECOND:

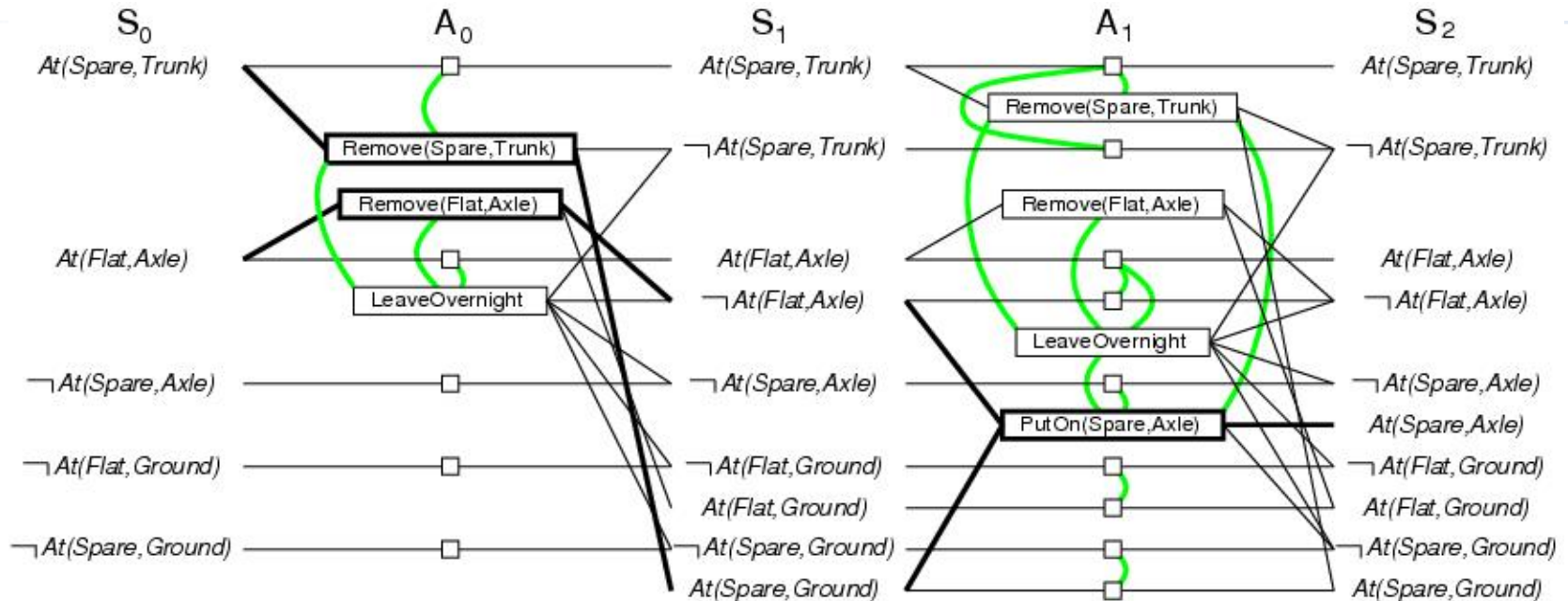
EFFECT:  $\neg \text{At}(\text{Spare}, \text{Ground}) \wedge \neg \text{At}(\text{Spare}, \text{Axle})$

$\wedge \neg \text{At}(\text{Spare}, \text{Trunk}) \wedge \neg \text{At}(\text{Flat}, \text{Ground}) \wedge \neg \text{At}(\text{Flat}, \text{Axle})$  )

Goal is to have a good spare tire properly mounted onto the car's axle,

Initial state has a flat tire on the axle and a good spare tire in the trunk.

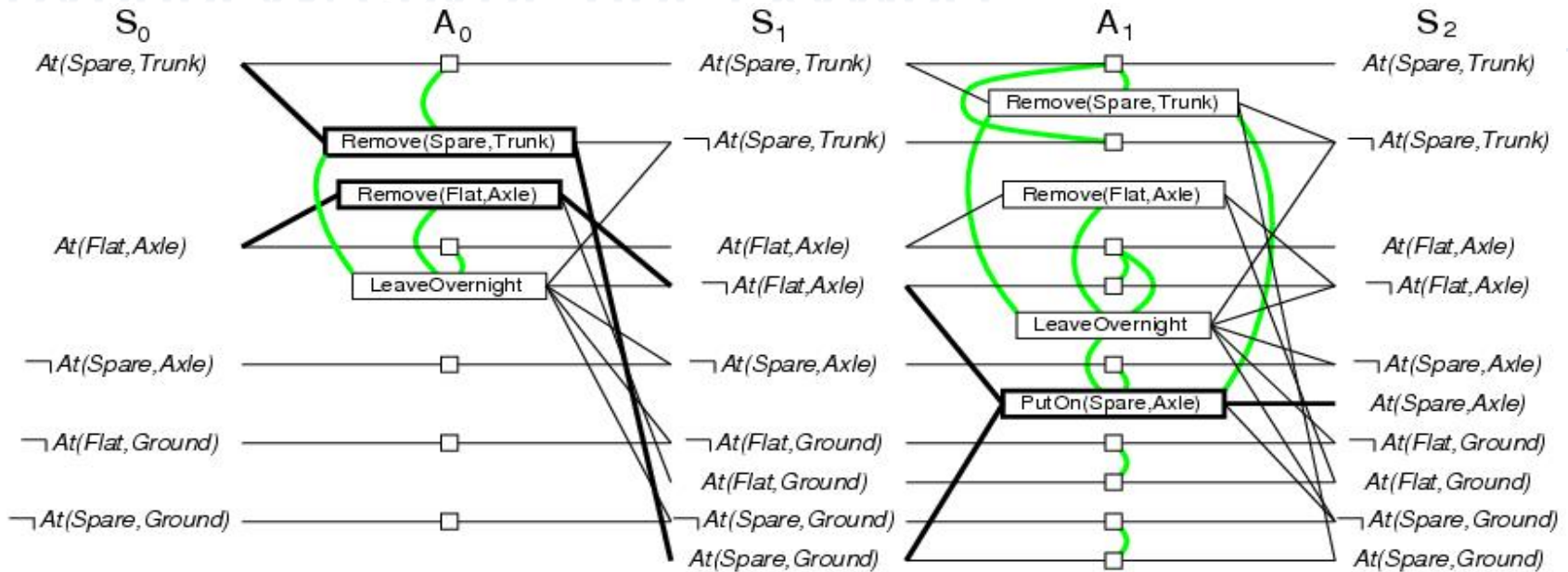
# GRAPHPLAN SPARE TIRE EXAMPLE



## Notes:

- This figure shows the *complete* Planning Graph for the problem
- Arcs show mutex relations (arcs between literals are *omitted* to avoid clutter)
- Omits unchanging positive literals (for example,  $Tire(Spare)$ )
- Omits irrelevant negative literals
- **Bold boxes & links** indicate the solution plan

# GRAPHPLAN SPARE TIRE EXAMPLE



- $S_0$  is initialized to 5 literals
  - from the problem initial state and the relevant negative literals
- no goal literal in  $S_0$  so **EXPAND-GRAPH add actions**
  - those with preconditions satisfied in  $S_0$
  - also adds *persistence actions* for literals in  $S_0$
  - adds the effects at level  $S_1$ , analyzes & adds mutex relations
- repeat until the goal is in level  $S_i$  or failure

```

Init( $At(Flat, Axle) \wedge At(Spare, Trunk)$ )

Goal( $At(Spare, Axle)$ )

Action(Remove(Spare, Trunk))
  PRECOND:  $At(Spare, Trunk)$ 
  EFFECT:  $\neg At(Spare, Trunk) \wedge At(Spare, Ground)$ )

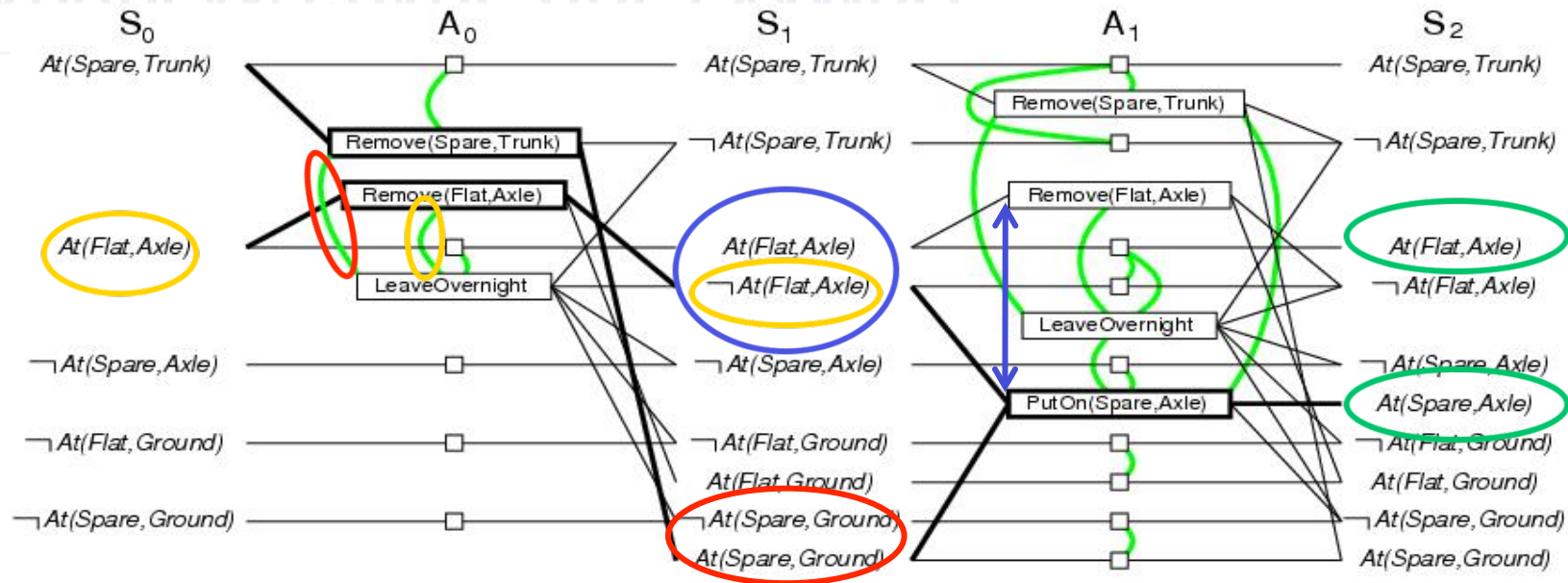
Action(Remove(Flat, Axle))
  PRECOND:  $At(Flat, Axle)$ 
  EFFECT:  $\neg At(Flat, Axle) \wedge At(Flat, Ground)$ )

Action(PutOn(Spare, Axle))
  PRECOND:  $At(Spare, Ground) \wedge \neg At(Flat, Axle)$ 
  EFFECT:  $At(Spare, Axle) \wedge \neg At(Spare, Ground)$ )

Action(LeaveOvernight)
  PRECOND:
  EFFECT:  $\neg At(Spare, Ground) \wedge \neg At(Spare, Axle)$ 
          $\wedge \neg At(Spare, Trunk) \wedge \neg At(Flat, Ground)$ 
          $\wedge \neg At(Flat, Axle)$ 
  
```



# GRAPHPLAN SPARE TIRE EXAMPLE



## EXPAND-GRAPH adds constraints: mutex relations

- inconsistent effects (action x vs action y)
  - Remove(Spare, Trunk) & LeaveOvernight:
  - At(Spare, Ground) & ¬At(Spare, Ground)
- interference (effect negates a precondition)
  - Remove(Flat, Axle) & LeaveOvernight:
  - At(Flat, Axle) as PRECOND & ¬At(Flat, Axle) as EFFECT
- competing needs (mutex preconditions)
  - PutOn(Spare, Axle) & Remove(Flat, Axle):
  - At(Flat, Axle) & ¬At(Flat, Axle)
- inconsistent support (actions to produce literals are mutex)
  - in S2, At(Spare, Axle) & At(Flat, Axle): only way to achieve At(Spare, Axle) is by PutOn(Spare, Axle) and that is mutex with the only action for obtaining At(Flat, Axle).

```

Init(At(Flat, Axle) ∧ At(Spare, Trunk))

Goal(At(Spare, Axle))

Action(Remove(Spare, Trunk))
  PRECOND: At(Spare, Trunk)
  EFFECT: ¬At(Spare, Trunk) ∧ At(Spare, Ground))

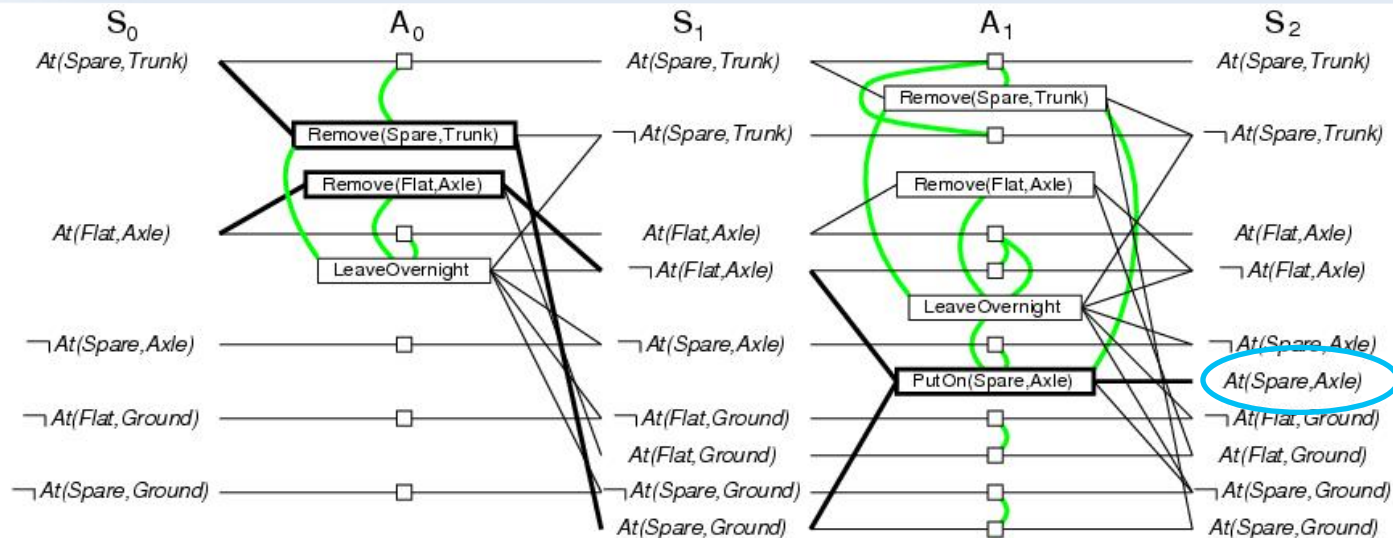
Action(Remove(Flat, Axle))
  PRECOND: At(Flat, Axle)
  EFFECT: ¬At(Flat, Axle) ∧ At(Flat, Ground))

Action(PutOn(Spare, Axle))
  PRECOND: At(Spare, Ground) ∧ ¬At(Flat, Axle)
  EFFECT: At(Spare, Axle) ∧ ¬At(Spare, Ground))

Action(LeaveOvernight)
  PRECOND:
  EFFECT: ¬At(Spare, Ground) ∧ ¬At(Spare, Axle)
         ∧ ¬At(Spare, Trunk) ∧ ¬At(Flat, Ground)
         ∧ ¬At(Flat, Axle) )
  
```



# GRAPHPLAN SPARE TIRE EXAMPLE



- In  $S_2$ , the goal literals exist, and they are not mutex with any other
  - Just 1 goal literal so obviously not mutex with any other goal
  - Since a solution may exist, EXTRACT-SOLUTION tries to find it
- EXTRACT-SOLUTION as backward search problem (other methods possible)
  - Initial state: last level of the PG,  $S_n$ , along with the goals from the planning problem
  - Actions from  $S_i$ 
    - Select any conflict-free actions in  $A_{i-1}$  with effects covering the goals
    - Conflict free = no 2 actions are mutex & no pair of their preconditions are mutex
  - Goal: Reach a state at level  $S_0$  such that all goals are satisfied
  - Cost: 1 for each action

# GRAPHPLAN SOLUTIONS

## × If EXTRACT-SOLUTION fails

- + At that point it records (level, goals) as a "*no-good*"
- + Subsequent calls can fail immediately if they require the same goals at that level

## × Complexity

- + We already know planning problems are computationally hard (PSPACE-complete)
  - × Require good heuristics
- + Heuristic for choosing an action at each level in backward search
  - Greedy search with level cost of literals
    - × 1. Pick literal with highest level cost
    - × 2. To achieve it, pick actions with easier preconditions
      - ★ Choose action with smallest sum (or max) of level costs for its preconds

- × Alternative to backward search for a solution
  - + EXTRACT-SOLUTION could formulate a Boolean CSP
    - × variables are actions at each level
    - × values are Boolean: an action is either *in* or *out* of the plan
    - × constraints are mutex relations & the need to satisfy each goal & pre condition

# GRAPHPLAN TERMINATION

GRAPHPLAN will in fact terminate and return failure when there is no solution.

- × Recall that **level off** means consecutive PG levels are identical
- × Now note that a graph may level off before a solution can be found, on a problem for which there is a solution
  - + Ex. Air Cargo: 1 plane and  $n$  pieces of cargo at airport A, all of which have airport B as their destination. Where only one piece of cargo can fit in the plane at a time.
    - × Graph levels off at level 4, from which full solution can't be extracted (that would require  $4n - 1$  steps)
- × We need to take account of the no-goods (goals that were not achievable) as well
  - + If it is possible that there might be fewer no-goods in the next level, then we should continue
- × Graph itself and the no-goods have both leveled off, with no solution found, we can terminate with failure

# GRAPHPLAN TERMINATION

- × Does GRAPHPLAN terminate?
- × Evidences that both graph and no-goods will level off
  - + Literals increase monotonically (and there are finite # of them)
    - × Once a literal appears, its persistence action causes it to stay
  - + Actions increase monotonically (and there are finite # of them)
    - × Once preconditions (literals) of an action appear at one level, they will appear at subsequent levels, and thus so will the action.
  - + Mutexes decrease monotonically
    - × Of 2 actions are mutex at  $A_i$ , they are also mutex at all previous levels where they appear
      - ★ The graph simplifying conventions may not show it
    - × Same holds for 2 literals
  - + No-goods decrease monotonically
    - × If a set of goals is not achievable at level  $i$ , they are not achievable at any previous level