Lecture 9

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Flow Networks

- A flow network $G=(V, E)$ is a directed graph in which each arc $(v_i, v_j) \in E$ has a nonnegative capacity $C_{ij}>0$; and if there is no arc from vertex v_i to vertex v_j, $C_{ij}=0$. Two vertices are distinguished in a flow network: a source vertex s and a sink vertex t. A flow in G is a real-valued function f: $V \times V \rightarrow R$.

- A flow network satisfies following properties:
- (1) Capacity constraint: for each (*u*, *v*) , $0 \le f(u,v) \le C(u,v)$.
- (2) Flow conservation: for $\forall u \in V - \{s,t\}$, $\sum_{x \in V} f(x,u) - \sum_{x \in V} f(u,x) = 0$.

- Given a flow network $G(V, E)$ with source vertex s and sink vertex t, and a flow f; we are normally required to find the maximum flow from the source s to the sink t. The key to the problem is to find an augmenting path p from s to t.

- Methods to find an augmenting path are as follow.
  - Depth-First Search (DFS);
  - Breadth-First Search (BFS);
  - Labelling Algorithm.

- Above algorithms add flow each time for an augmenting path. If the maximum flow is a, and the time finding an augmenting path is m, the time complexity for computing the maximum flow is $O(a*m)$.

Power Network

- **Source: ACM Southeastern Europe 2003**
- **IDs for Online Judge: POJ 1459, ZOJ 1734, UVA 2760**

- SPFA algorithm can be used to calculate the augmenting path from the source vertex *st* to the sink vertex *en*.
- Flows are as edges' weights. The shortest path from the source vertex *st* to the sink vertex *en* can be calculated. The path is the augmenting path. Suppose *h* is the queue, pointers for the front and rear for *h* are *l* and *r* respectively; and *pre* is the precursor pointer in the augmenting path.

Trash

- **IDs for Online Judge: Ural 1076**