Dynamic Sculpting of Triangular NURBS Objects (Extended Abstract)

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Among geometric representation schemes, non-uniform rational B-splines (NURBS) have become an industry standard mainly because they are a unified representation of both free-form shapes and standard analytic shapes. The main drawback of tensor product NURBS, however, is that the surface patches are rectangular. Consequently, the designer is forced to model multisided irregular shapes using degenerate patches with deteriorated inter-patch continuity. To compensate, explicit non-linear smoothness constraints must be enforced within the underlying rational representation, thus increasing the complexity of the design task in general.

Triangular B-splines [1] have recently emerged as a promising CAGD tool. This is primarily because they can represent complex non-rectangular shapes over arbitrary triangulated domains with low degree piecewise polynomials that nonetheless maintain relatively high-order continuity. For instance, we can construct C^1 continuous surfaces with quadratic triangular B-splines, whereas C^1 tensor-product splines must be at least biquadratics. Triangular B-splines retain a considerable breadth of geometric coverage. They can express smooth non-rectangular shape without degeneracy. They can also model discontinuities by varying the knot distribution. They subsume Bernstein-Bezier triangles as a special case with n-fold knots. Moreover, any piecewise polynomial can be represented as a linear combination of triangular B-splines [4]. Thus, triangular B-splines can serve as the canonical representation of surfaces with irregular parametric domains.

In this paper, we propose triangular NURBS, the rational generalization of triangular B-splines, with weights as extra degrees of freedom to increase the power of the modeling scheme. As in conventional NURBS, fixing the weights to unity reduces the model to dynamic triangular B-splines. Using triangular NURBS, the designer can overcome the limitations of tensor product NURBS. However, conventional geometric design with triangular NURBS can be problematic for the following reasons:

- Normally the designer exercises geometric control by selecting knots and adjusting control points and weights. Despite the variety of interaction devices, this “indirect” design process can be especially laborious for triangular splines, because of the irregularity of control points and knot vectors.
- Relevant design requirements are shape oriented and not control point and weight oriented. Because of the geometric “redundancy” of rational models, indirect shape refinement remains *ad hoc* and ambiguous.
- Typical design requirements may be posed in both quantitative and qualitative terms. Therefore, it can be very frustrating with indirect design to shape, say, a “fair” surface that approximates unstructured 3D data.

To ameliorate the indirect design process, we furthermore develop a physics-based generalization of triangular NURBS using Lagrangian mechanics and finite elements. The new surface model, *dynamic triangular NURBS*, combines the geometric features of triangular NURBS with the demonstrated conveniences of interaction within a physical dynamics framework. Some of the advantages of physics-based shape design are as follows:

- Shape design is generally a time-varying process—a designer is often interested not only in the final shape but also in the intermediate shape variation due to parameter changes. Since time is fundamental to the physics-based formulation, dynamic models can continuously evolve control points and weights in response to applied forces to produce meaningful shape variation.
- The behavior of the dynamic model is governed by physical laws. Through a computational physics simulation, the model responds dynamically to applied simulated forces in a natural and predictable way. Shapes can be sculpted interactively using a variety of force-based “tools.”
- Functional design requirements can be readily implemented as deformation (fairness) energies and geometric constraints. As a dynamic model reaches equilibrium, it acts as a nonlinear shape optimizer subject to the imposed constraints.

- A physics-based framework based upon the rational geometric model permits appropriate weight values to be determined automatically in accordance with various physical parameters and geometric requirements.
- The physical model is built upon a standard geometric foundation. While shape design may proceed interactively or automatically at the physical level, existing geometric toolkits are concurrently applicable at the geometric level.

Like dynamic tensor product NURBS (D-NURBS) [5], the control points and weights of the geometric triangular NURBS become generalized (physical) coordinates in the dynamic model. We introduce time, mass, and deformation energy into triangular NURBS and employ Lagrangian dynamics to formulate their motion equations. We then use finite element analysis [2] to reduce these equations to efficient algorithms that can be simulated using standard numerical methods [3]. Time is fundamental to the dynamic formulation, which can continuously evolve the control points and weights in response to applied forces to produce physically meaningful and intuitively predictable shape variation.

We have implemented interactive software for dynamic triangular B-splines (the special case with unit weights) using iterative numerical methods. Our prototype software demonstrates the flexibility of the dynamic models across several applications, including interactive sculpting through forces and physical parameters, the fitting of unstructured data, and solid rounding with geometric and physical constraints. Our results indicate that dynamic triangular NURBS provide a systematic and unified approach for a variety of modeling tasks.

Because NURBS have been incorporated into a number of industry standards such as IGES, PHIGS+, openGL, and STEP, our research can serve as a basis for future work on physics-based or constraint-based paradigms in well established CAGD methodologies. With the advent of high-performance graphics systems, the physics-based framework is readily incorporated into commercial design systems to interactively model and sculpt complex shapes in real-time.

References

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