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Diffusion-Driven High-Order Matching of Partial Deformable Shapes

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Abstract

This paper tackles the matching problem of partial deformable shapes with changing boundary and varying topology. We compute high-order graph matching directly on manifolds, without global/local surface parameterization. In particular, we articulate the heat kernel tensor (HKT), which is a high-order potential of geometric compatibility between feature tuples measured by heat kernels within bounded time. Inherited from the heat kernel, the HKT is multi-scale, invariant to isometric deformation, resilient to noise, and robust to topology changes. We also build up a two-level hierarchy via feature clustering, by which the searching space of HKT is greatly reduced. To evaluate the proposed method, we conduct experiments in various aspects, including scale, noise, deformation, comparison, and semantic matching.

1. Introduction and related work

Matching 3D deformable shapes remains a challenging problem with growing interest in computer vision, which affords the enabling technique for shape comparison, retrieval, and recognition. The analysis of dynamic shapes relies on correct correspondences between them, where shape matching is a fundamental and important tool. To reduce the complexity and overcome the ambiguity, we refer shape matching to finding sparse correspondences of representative points (i.e., features) on shapes in this paper.

One challenge of shape matching is the nonrigid deformation. Dynamic shapes acquired from natural objects such as face, human body, and animal, may undergo large deformation. Existing methods tackle this problem by various geometric tools, including geodesics, conformal geometry, shape spectrum, and spectral embedding. In this paper, we turn our foci to some other technical challenges that have been under-explored by previous work. First, dynamic scans are oftentimes *partial* shapes with possible boundary changes. Some techniques based on global geometry, such as shape-DNA [5], global point signature [7], and surface parameterization [10], are therefore not suitable for this case. Second, the *outlier* problem is a well-known challenge for feature matching, which is unavoidable for partial shapes. Outliers are points that do not have proper matches on the other shape, resulting from missing information or incomplete feature detection. Consequently, the matching task cannot be fulfilled by simply finding the most similar feature-pair (i.e. the nearest-neighbor search [2, 3]). Finally, *computational efficiency* is required throughout the entire pipeline of feature detection and matching. Massive data analysis demands faster algorithms that are able to process dynamic shapes efficiently.

Our idea to tackle the aforementioned challenges is motivated by two promising and powerful tools: diffusion geometry and high-order graph matching. A 3D shape without self-intersection is a 2-manifold embedded in $\mathbb{R}^3$. In diffusion geometry, eigenvalues and eigenfunctions of the Laplace-Beltrami operator on manifolds have been widely applied to shape matching [5, 6, 7, 8]. As discussed above, they are global characteristics, subject to boundary changes. Heat kernel, also resulting from the eigen-system, has demonstrated its utility in shape matching [3]. It has many attractive properties, including multi-scale, isometric invariance, and robustness to noise. Most importantly, it is localized on manifolds. With bounded time, it is mostly determined by the local geometry, which affords its applications in partial shape matching.

High-order graph matching [1, 9], has demonstrated its superior performance of combating outliers and mismatches. It considers geometric compatibility between feature tuples, and matches them collectively as a graph. However, to apply graph matching on deformable shapes is not trivial. The previous idea is to compulsorily flatten the shape to a 2D "image" via surface parameterization [10], which has encountered many difficulties in handling flexible boundary, topological change, large distortion, bad triangulation, and model cutting. It is highly desirable and necessary to explore the graph matching directly on manifolds, with resilience to both boundary and topology changes.

In this paper, we develop a diffusion-driven method for matching partial deformable shapes with possible boundary and topology changes, by conducting high-order graph matching directly on manifolds. We propose the heat kernel tensor (HKT), which is a high-order potential of geometric compatibility of feature tuples on manifolds. To facilitate the matching process, we further build up a two-level hierarchy via feature clustering. This simple hierarchy greatly reduces the search space of HKT, and therefore the computation time. Through various experiments, we demonstrate this method can match partial shapes of deforming objects, as well as similar shapes from different objects by producing semantic correspondences.

2. Heat kernel tensor

Geometric relations among features are extremely important on deformable shapes, and collectively they are much more reliable than single feature point in shape matching. Therefore, we adopt the advanced tensor matching [1], and transplant it to manifolds via a diffusion-driven relation measure, given by

$$d_t(x, y) = \frac{1}{4}(-t \log h_t(x, y))^{1/2}. \quad (1)$$

Here, $h_t(x, y)$ denotes the heat kernel from point x to y at time t

$$h_t(x, y) = \sum_{0}^{\infty} e^{-\lambda_l t} \phi_l(x) \phi_l(y), \quad (2)$$

where λ_l and ϕ_l are the l-th eigenvalue and eigenfunction of the Laplace-Beltrami operator. When $t \to 0$, $d_t(x, y)$ is indeed a metric and converges to the geodesic between x and y.

We consider two partial shapes M_1 and M_2 with overlaps and boundary changes. Let N_1 be the number of features extracted on M_1, and N_2 be the one on M_2. A pair $i = (i_1, i_2)$ denotes a candidate match with a point i_1 from M_1 and i_2 from M_2. The problem of matching point sets is equivalent to finding an assignment matrix $X_{N_1 \times N_2}$, such that

$$X_{i_1,i_2} = \begin{cases} 1 & i_1 \text{ matches } i_2 \\ 0 & \text{otherwise} \end{cases}, \text{with} \sum_{i_2} X_{i_1,i_2} \leq 1. \quad (3)$$

Note that there may be outliers in the feature set. As shown in Fig. 1, some outliers are circled. For an outlier i_1, there is no match in the second feature set, i.e., $\sum_{i_2} X_{i_1,i_2} = 0$. We adopt the tensor formulation [1] for high-order graph matching on manifold. Specifically, we consider a tuple of three candidate matches (i, j, k) without conflicts, i.e., $i_1 \neq j_1 \neq k_1$ and $i_2 \neq j_2 \neq k_2$. They may form two "triangles" by connecting them with d_t, as shown in Fig. 1. Since small heat kernels are error-prone, we select large heat kernels with a threshold $\epsilon_h(t) = 10^{-6}$. In the case when the three points do not form a triangle, we simply drop this tuple.

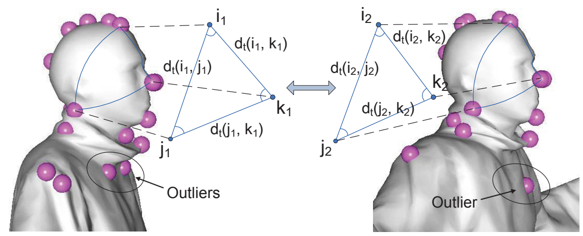

Figure 1. HKT for shape matching. Three candidate matches (i, j, k) form two "triangles". Some outlier features are circled.

The tuple of candidate matches is then embedded into a 3D space by three angles of this triangle. The distance in the embedded space is given by

$$d_\theta(i, j, k) = \|\theta_{i_1,j_1,k_1} - \theta_{i_2,j_2,k_2}\|_2, \quad (4)$$

where θ_{i_1,j_1,k_1} is a vector comprising three angles of the triangle formed by points i_1, j_1, k_1, and $\|.\|_2$ denotes the l^2-norm. The affinity of the tuple (i, j, k) without conflicts is defined as

$$\tau_{i,j,k} = e^{-d_\theta(i,j,k)^2/\sigma}, \quad (5)$$

where σ is a parameter, which can be set as $\sigma = \mathrm{mean}(d_\theta)$. For tuples with conflicts, we let their affinities equal to zero. The high-order score of assignment X is defined as

$$\text{score}(X) = \sum_{i,j,k} \tau_{i,j,k} X_{i_1,i_2} X_{j_1,j_2} X_{k_1,k_2}. \quad (6)$$

We rewrite the score using tensor notation, given by

$$\text{score}(X) = T \otimes_1 X \otimes_2 X \otimes_3 X, \quad (7)$$

where $\otimes_d$ denotes the tensor product in d dimension. We call T the *heat kernel tensor*, as it utilizes heat kernels to form the tensor. The HKT can be fused with different order of potentials. Here, the HKT is a $3rd$-order tensor with entries $\tau_{i,j,k}$ defined in Eq. (5). The final results are obtained according to their matching scores subject to conflict constraints in Eq. (3).

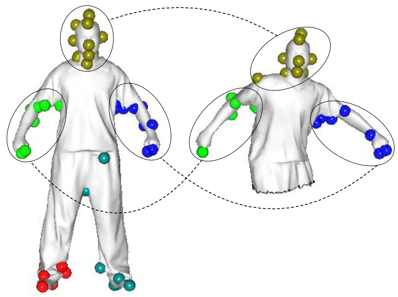

Figure 2. Matching hierarchy. Extracted features are clustered into sub-graphs.

3. Matching hierarchy

For articulated shapes, we design a two-level hierarchy to improve the time performance by reducing the searching space. Articulated shapes with long branches can be easily segmented using some low frequency eigenfunctions of the Laplace-Beltrami operator [4]. In the upper level, we find centers of clusters as the local extrema of the first two non-trivial Laplace-Beltrami eigenfunctions, and remove redundant ones that are very close to selected centers. In the lower level, extracted shape features are then clustered into sub-graphs based on their heat kernels to the cluster centers. The goal of the upper-level matching is to reduce the searching space, and it can be skipped whenever necessary.

The cluster centers comprise a hyper-graph in the upper level of the hierarchy, as shown in Fig. 2. In the hyper-graphs with hyper-nodes (cluster centers), we compute their HKT. We release conflicting constraints by allowing candidate matches that have matching scores greater than 80% of the maximal one. This will prune diverse sub-graphs, and reduce the search space of HKT. At the lower level, we run HKT in each cluster. For the high-order optimization in Eq. (7), we use the tensor power iteration with l^1-norms of columns. The complexity of one power iteration is $O(m)$, where m is the number of non-zero elements in the tensor. We restrict the number of triangles (i.e., non-zero elements) to $64N_1$ by randomly selecting tuples. As a result, the computation of HKT is very efficient.

4. Experimental results

To evaluate the proposed method, we conduct various experiments, including scale change (Fig. 3), noise/topology change (Fig. 4)), large deformation (Fig. 5), and semantic matching (Fig. 6). The only parameter of our method needed to be tuned is time t in HKT. We set $t = 20$ for two-level matching and $t = 60$ for one-level matching.

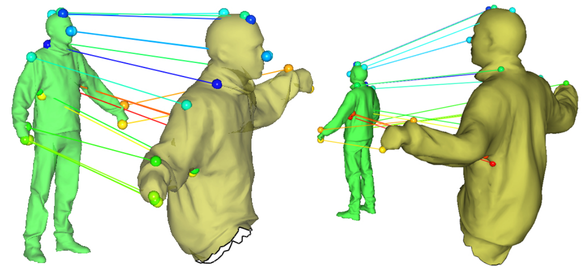

Figure 3. Experiment of deforming shapes with scale changes by our method.

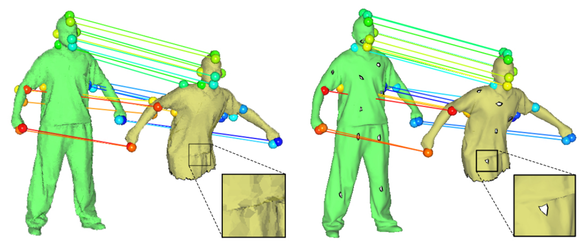

Figure 4. Matching with noise (*Left*) and topology changes (*Right*).

In Fig. 4, we add Gaussian noise ($\sigma = 10\%$ of average edge length) to vertex coordinates, and also introduce topology noise by punching holes on models. For large deformation, we test our method through a sequence of deforming shapes of a dancing woman, with selected frames shown in Fig. 5. The matching results are stable under large deformations across these frames. Our method can also be applied to match partial shapes from similar objects, resulting in semantic correspondences shown in Fig. 6. We skip the upper-level matching, since we may need geometric constraints from faraway points. The fists of a man are matched to the wrists of a woman. This is because the woman's arms are thinner than the man's arms, thus, the heat diffuses faster on the woman's arms. And the first hand is a closed surface, while the second one is open.

For quantitative evaluation, we compute differences of geodesics between matched pairs. For a match pair $i = (i_1, i_2)$ in the correspondence set S, we find $j = \min_j(d_\mu(i_1, j_1))$ with geodesic d_μ, and compute

$$\text{error}(i) = \frac{|d_\mu(i_1, j_1) - d_\mu(i_2, j_2)|}{\bar{e}}, \qquad (8)$$

where $\bar{e}$ denotes the average edge length. The mean errors of all correspondences are documented in Tab. 1.

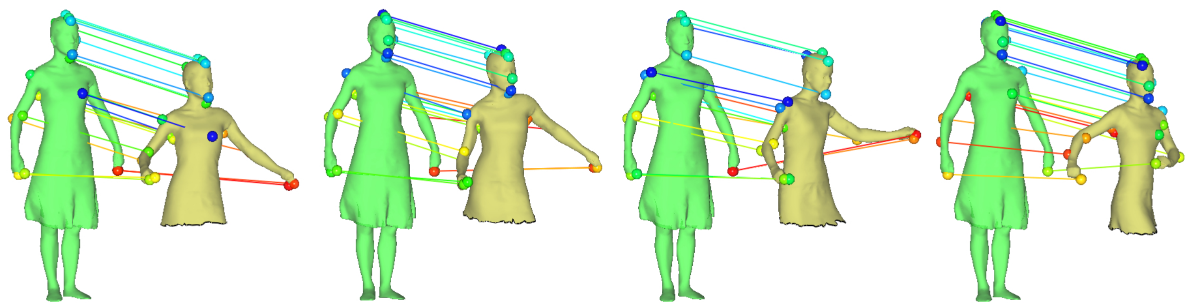

Figure 5. Selected frames of matching largely deforming objects.

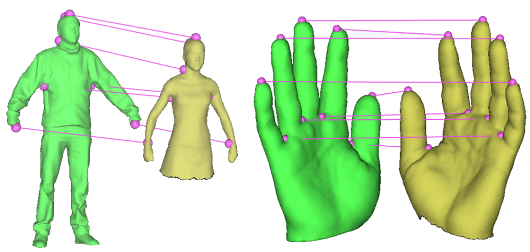

Figure 6. Matching similar objects.

Data	$\#V_1, \#V_2$	Time (sec)		Mean error
		HKS	HKT	
man1	10.0k, 5.7k	4.99	1.30	0.788*
man2	10.0k, 5.9k	3.14	0.5	0.703*
woman	10.0k, 5.8k	5.59	0.81	0.822
hand	10.0k, 9.0k	1.45	0.16	N/A

* evaluated before scaling or noise

Table 1. Time performance and quantitative evaluation of our method.

We also detail the time performance running on a laptop. We adopt the HKS for feature detection, which shares the computation of eigen-decomposition with the HKT. It may be noted that, the computation of eigen-decomposition is not listed in Tab. 1. It costs about 45 seconds for a pair of meshes by our implementation.

5 Conclusion

We have detailed a new method for matching partial deformable shapes equipped with diffusion geometry and high-order graph matching. The technical core is found upon the HKT, which is a diffusion-driven representation of high-order geometric compatibility. Without the need of any surface flattening or metric modification via surface parameterization, our method directly computes the high-order graph matching on manifolds. All the merits and versatile applications using our method are demonstrated through our extensive experiments, and validated in various aspects. Upon correct matching of partial deformable shapes, our ongoing and near-future research efforts are geared towards dense registration of partial non-rigid shapes, and space-time data completion from incomplete data input.

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