Brain Surface Parameterization Using Riemann Surface Structure

Yalin Wang[1], Xianfeng Gu[2], Kiralee M. Hayashi[3], Tony F. Chan[1], Paul M. Thompson[3], and Shing-Tung Yau[4]

[1] Mathematics Department, UCLA, Los Angeles, CA 90095, USA
[2] Comp. Sci. Department, SUNY at Stony Brook, Stony Brook, NY 11794, USA
[3] Lab. of Neuro Imaging, UCLA School of Medicine, Los Angeles, CA 90095, USA
[4] Department of Mathematics, Harvard University, Cambridge, MA 02138, USA
{ylwang, chan}@math.ucla.edu, gu@cs.sunysb.edu,
{khayashi, thompson}@loni.ucla.edu, yau@math.harvard.edu

Abstract. We develop a general approach that uses holomorphic 1-forms to parameterize anatomical surfaces with complex (possibly branching) topology. Rather than evolve the surface geometry to a plane or sphere, we instead use the fact that all orientable surfaces are Riemann surfaces and admit conformal structures, which induce special curvilinear coordinate systems on the surfaces. Based on Riemann surface structure, we can then canonically partition the surface into patches. Each of these patches can be conformally mapped to a parallelogram. The resulting surface subdivision and the parameterizations of the components are intrinsic and stable. To illustrate the technique, we computed conformal structures for several types of anatomical surfaces in MRI scans of the brain, including the cortex, hippocampus, and lateral ventricles. We found that the resulting parameterizations were consistent across subjects, even for branching structures such as the ventricles, which are otherwise difficult to parameterize. Compared with other variational approaches based on surface inflation, our technique works on surfaces with arbitrary complexity while guaranteeing minimal distortion in the parameterization. It also offers a way to explicitly match landmark curves in anatomical surfaces such as the cortex, providing a surface-based framework to compare anatomy statistically and to generate grids on surfaces for PDE-based signal processing.

1 Introduction

In brain imaging research, parameterization of various types of anatomical surface models in magnetic resonance imaging (MRI) scans of the brain involves computing a smooth (differentiable) one-to-one mapping of regular 2D coordinate grids onto the 3D surfaces, so that numerical quantities can be computed easily from the resulting models [1,2]. Even so, it is often difficult to smoothly deform a complex 3D surface to a sphere or 2D plane without substantial angular or area distortion. Here we present a new method to parameterize brain surfaces based on their Riemann surface structure. By contrast with variational

J. Duncan and G. Gerig (Eds.): MICCAI 2005, LNCS 3750, pp. 657–665, 2005.

approaches based on surface inflation, our method can parameterize surfaces with arbitrary complexity including branching surfaces not topologically homeomorphic to a sphere (higher-genus objects) while formally guaranteeing minimal distortion.

1.1 Previous Work

Brain surface parameterization has been studied intensively. Schwartz et al. [3], and Timsari and Leahy [4] compute quasi-isometric flat maps of the cerebral cortex. Hurdal and Stephenson [5] report a discrete mapping approach that uses circle packings to produce "flattened" images of cortical surfaces on the sphere, the Euclidean plane, and the hyperbolic plane. Angenent et al. [6] represent the Laplace-Beltrami operator as a linear system and implement a finite element approximation for parameterizing brain surfaces via conformal mapping. Gu et al. [7] propose a method to find a unique conformal mapping between any two genus zero manifolds by minimizing the harmonic energy of the map.

1.2 Theoretical Background and Definitions

We begin with some formal definitions that will help to formulate the parameterization problem(for further reading, please refer to [8]). For a manifold M with an atlas $\mathcal{A} = \{U_\alpha, \phi_\alpha\}$, if all chart transition functions $\phi_{\alpha\beta} = \phi_\beta \circ \phi_\alpha^{-1} : \phi_\alpha(U_\alpha \cap U_\beta) \to \phi_\beta(U_\alpha \cap U_\beta)$ are holomorphic, $\mathcal{A}$ is a conformal atlas for M. A chart $\{U'_\alpha, \phi'_\alpha\}$ is *compatible* with an atlas $\mathcal{A}$, if the union $\mathcal{A} \cup \{U'_\alpha, \phi'_\alpha\}$ is still a conformal atlas. Each conformal compatible equivalence class is a conformal structure. A 2-manifold with a conformal structure is called a *Riemann surface*. It has been proven that all metric orientable surfaces are Riemann surfaces.

Holomorphic and meromorphic functions and differential forms can be generalized to Riemann surfaces by using the notion of conformal structure. For example, a *holomorphic one-form* ω is a complex differential form, such that in each local frame $z_\alpha = (u_\alpha, v_\alpha)$, the parametric representation is $\omega = f(z_\alpha)dz_\alpha$, where $f(z_\alpha)$ is a holomorphic function. On a different chart $\{U_\beta, \phi_\beta\}$, $\omega = f(z_\alpha(z_\beta))\frac{dz_\alpha}{dz_\beta}dz_\beta$. For a genus g closed surface, all holomorphic one-forms form a real $2g$ dimensional linear space.

At a *zero point* $p \in M$ of a holomorphic one-form ω, any local parametric representation $\omega = f(z_\alpha)dz_\alpha, f|_p = 0$. According to the Riemann-Roch theorem, in general there are $2g - 2$ zero points for a holomorphic one-form defined on a surface of genus g.

A holomorphic one-form induces a special system of curves on a surface, the so-called *conformal net*. A curve $\gamma \subset M$ is called a horizontal trajectory of ω, if $\omega^2(d\gamma) \geq 0$; similarly, γ is a vertical trajectory if $\omega^2(d\gamma) < 0$. The horizontal and vertical trajectories form a web on the surface. The trajectories that connect zero points, or a zero point with the boundary are called *critical trajectories*. The critical horizontal trajectories form a graph, which is called the *critical graph*. In general, the behavior of a trajectory may be very complicated, it may have infinite length and may be dense on the surface. If the critical graph is finite, then

all the horizontal trajectories are finite. The critical graph partitions the surface into a set of non-overlapping patches that jointly cover the surface, and each patch is either a topological disk or a topological cylinder. Each patch $\Omega \subset M$ can be mapped to the complex plane using the following formulae. Suppose we pick a base point $p_0 \in \Omega$, and any path γ that connects p_0 to p. Then if we define $\phi(p) = \int_\gamma \omega$, the map ϕ is conformal, and $\phi(\Omega)$ is a parallelogram. We say ϕ is the conformal parameterization of M induced by ω. ϕ maps the vertical and the horizontal trajectories to iso-u and iso-v curves respectively on the parameter plane. The structure of the critical graph and the parameterizations of the patches are determined by the conformal structure of the surface. If two surfaces share similar topologies and geometries, they can support consistent critical graphs and segmentations (i.e. surface partitions), and the parameterizations are consistent as well. Therefore, by matching their parameter domains, the entire surfaces can be directly matched in 3D. This generalizes prior work in medical imaging that has matched surfaces by computing a smooth bijection to a single canonical surface, such as a sphere or disk.

This paper takes the advantage of conformal structures of surfaces, consistently segments them and parameterizes the patches using a holomorphic 1-form. We call this process - i.e., finding a critical graph and partitioning the surface into conformally parameterized patches - the *holomorphic flow segmentation*. This parameterization and partitioning of the surface is completely determined by the surface geometry and the choice of the holomorphic 1-form. (Note that this differs from the typical meaning of segmentation in medical imaging, and is concerned with the segmentation, or partitioning, of a general surface, rather than classification of voxels in an image). Computing holomorphic 1-forms is equivalent to solving elliptic differential equations on surfaces, and in general, elliptic differential operators are stable. Therefore the resulting surface segmentations and parameterizations are intrinsic and stable, and are applicable for matching noisy surfaces derived from medical images.

2 Holomorphic Flow Segmentation

To compute the holomorphic flow segmentation of a surface, first we compute the conformal structure of the surface; then we select one holomorphic differential form, and locate the zero points on it. By tracing horizontal trajectories through the zero points, the critical graph can be constructed and the surface is divided into several patches. Each patch can then be conformally mapped to a planar parallelogram by integrating the holomorphic differential form.

In our work, surfaces are represented as triangular meshes, namely piecewise polygonal surfaces.The computations with differential forms are based on solving elliptic partial differential equations on surfaces using the finite element method.

2.1 Conformal Structures Computation

A method to compute the conformal structure of a surface was introduced in [9]. Suppose M is a closed genus $g > 0$ surface with a conformal atlas $\mathcal{A}$. The con-

formal structure $\mathcal{A}$ induces holomorphic 1-forms; all holomorphic 1-forms form a linear space $\Omega(M)$ of dimension $2g$ which is isomorphic to the first cohomology group of the surface $H^1(M, \mathcal{R})$. The set of holomorphic one-forms determines the conformal structure.

2.2 Canonical Conformal Parameterization Computation

Given a Riemann surface M, there are infinitely many holomorphic 1-forms, but each of them can be expressed as a linear combination of the basis elements. We define a canonical conformal parameterization as any linear combination of the set of holomorphic basis functions ω_i, $i = 1, ..., g$. They satisfy $\int_{\zeta_i} \omega_j = \delta_i^j$, where $\zeta_i, i = 1, ...n$ are homology bases and δ_i^j is the Kronecker symbol. Then we compute a *canonical conformal parameterization* $\omega = \sum_{i=1}^{n} \omega_i$.

2.3 Zero Points Location

For surface with genus $g > 1$, any holomorphic 1-form ω has $2g - 2$ zero points. The horizontal trajectories through the zero points will partition the surface into several patches. Each patch is either a topological disk or a cylinder, and can be conformally parameterized by ω using $\phi(p) = \int_\gamma \omega$.

Estimating the Conformal Factor. Suppose we already have a global conformal parameterization, induced by a holomorphic 1-form ω. Then we can estimate the conformal factor at each vertex, using the following formulae: $\lambda(v) = \frac{1}{n} \sum_{[u,v] \in K_1} \frac{|\omega([u,v])|^2}{|r(u) - r(v)|^2}, u, v \in K_0$, where n is the valence of vertex v.

Locating Zero Points. We find the cluster of vertices with relatively small conformal factors (the lowest $5 - 6\%$). These are candidates for zero points. We cluster all the candidates using the metric on the surface. For each cluster, we pick the vertex that is closest to the center of gravity of the cluster, using the surface metric to define geodesic distances.

2.4 Holomorphic Flow Segmentation

Tracing Horizontal Trajectories. Once the zero points are located, the horizontal trajectories through them can be traced. First we choose a neighborhood U_v of a vertex v representing a zero point, U_v is a set of neighboring faces of v, then we map it to the parameter plane by integrating ω. Suppose a vertex $w \in U_v$, and a path composed by a sequence of edges on the mesh is γ, then the parameter location of w is $\phi(w) = \int_\gamma \omega$.

The map $\phi(w)$ is a piecewise linear map. Then the horizontal trajectory is mapped to the horizontal line $y = 0$ in the plane. We slice $\phi(U_v)$ using the line $y = 0$ by edge splitting operations. Suppose the boundary of $\phi(U_v)$ intersects $y = 0$ at a point v', then we choose a neighborhood of v' and repeat the process. Each time we extend the horizontal trajectory and encounter edges intersecting the trajectory, we insert new vertices at the intersection points, until the trajectory reaches another zero point or the boundary of the mesh. We repeat the tracing process until each zero point connects 4 horizontal trajectories.

Critical Graph. Given a surface M and a holomorphic 1-form ω on M, we define the graph $G(M, \omega) = \{V, E, F\}$, as the critical graph of ω. Here V is the set of zero points of ω, E is the set of horizontal trajectories connecting zero points or the boundary segments of M, and F is the set of surface patches segmented by E.

Given two surfaces with similar topologies and geometries, by choosing appropriate holomorphic 1-forms, we can obtain isomorphic critical graphs, which will be used for patch-matching described in the next section.

3 Experimental Results

We tested our algorithm on various anatomic surfaces extracted from 3D MRI scans of the brain to illustrate the approach.

Figure 1 (a)-(d) shows experimental results for a hippocampal surface, a structure in the medial temporal lobe of the brain. The original surface is shown in (a). (b) shows the conformal mapping of (a) to a sphere with a variational method introduced in [7]. Since the shape of hippocampal surface is not quite similar to a sphere, lots of distortion has been introduced. In our method, we leave two holes on the front and back of the hippocampal surface, representing its anterior junction with the amygdala, and its posterior limit as it turns into the white matter of the fornix. It can be logically represented as an open boundary genus one surface, a cylinder (note that spherical harmonic representations would also be possible, if the ends were closed). The computed conformal structure is shown in (c). Then we can conformally map the hippocampus to a rectangle (d). Since the surface of rectangle is similar to the one of hippocampus, the detailed surface information is well preserved in (d). Compared with other spherical parameterization methods (e.g. (b)), which may have high-valence nodes and dense tiles at the poles of the spherical coordinate system, our parameterization can represent the surface with minimal distortion.

Shape analysis of the lateral ventricles is of great interest in the study of psychiatric illnesses, including schizophrenia, and in degenerative diseases such as Alzheimer's disease. These structures are often enlarged in disease and can provide sensitive measures of disease progression. We can optimize the conformal parameterization by topology modification. For the lateral ventricle surface in each brain hemisphere, we introduce five cuts. Since these cutting positions are at the end of the frontal, occipital, and temporal horns of the ventricles, they can potentially be located automatically. The second row in Figure 1 shows 5 cuts introduced on three subjects ventricular surfaces. After the cutting, the surfaces become open boundary genus 4 surfaces.

Figure 1 (e)-(g) show parameterizations of the lateral ventricles of the brain. (e) shows the results of parameterizing a ventricular surface for a 65-year-old patient with HIV/AIDS (note the disease-related enlargement), (f) the results for the ventricular model of a 21-year-old control subject, and (g) the results for a 28-year-old control subject. The surfaces are initially generated by using an unsupervised tissue classifier to isolate a binary map of the cerebrospinal fluid in

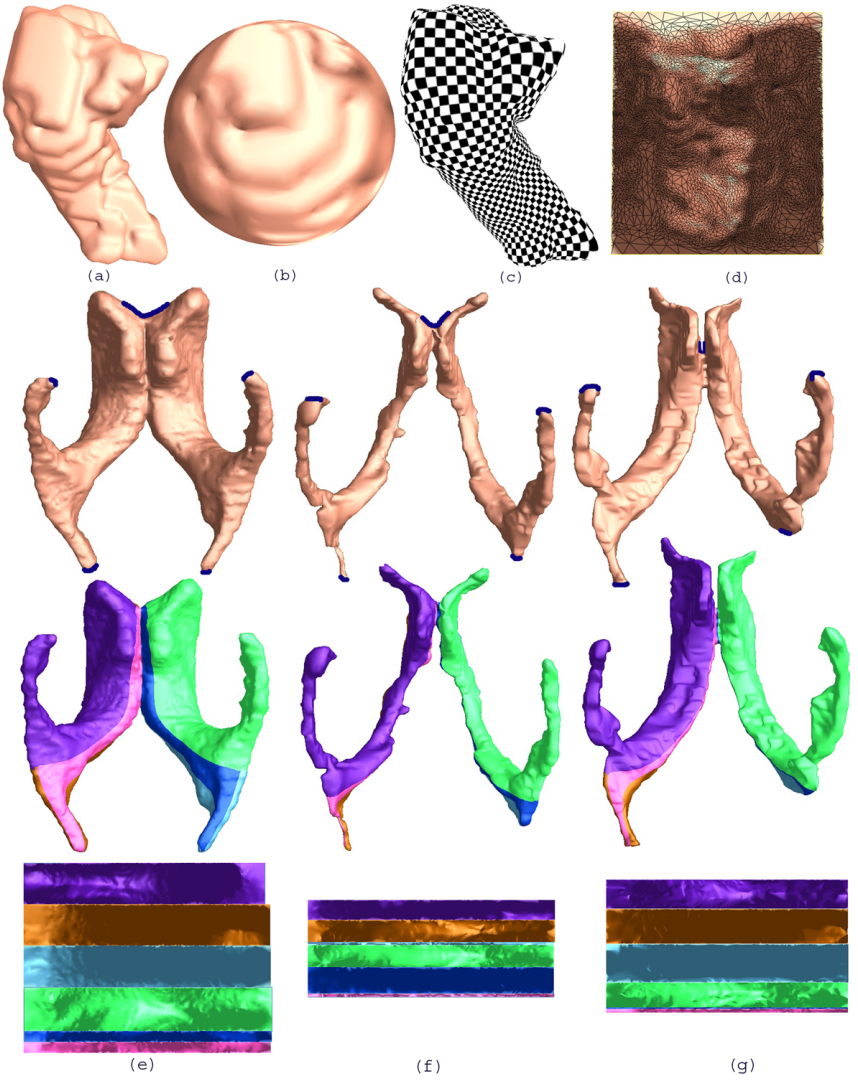

Fig. 1. Illustrates surface parameterization results for the hippocampal surface and the lateral ventricles. (a) is the original hippocampal surface; (b) the result of inflation of surface (a) to a sphere; (c) the computed conformal structure; and (d) the rectangle that (a) is conformally mapped to. The second row shows how 5 cuts are introduced; they convert the lateral ventricle surface into a genus 4 surface. (e)-(g) show models parameterized using holomorphic 1-forms, for a 65-year- old subject with HIV/AIDS, a healthy 21-year-old subject and a second healthy 28-year-old subject, respectively. The computed holomorphic flow segmentations and their associated sets of rectangular parameter domains are shown (the texture mapped into the parameter domain here simply corresponds to the intensity of the surface rendering, which is based on the surface normals).

the MR image, and tiling the surface of the largest connected component inside the brain. Based on the computed conformal structure, we can partition the surface into 6 patches. Each patch can be conformally mapped to a rectangle. Although the three brain ventricle shapes are very different, the segmentation results are consistent in that the surfaces are partitioned into patches with the same relative arrangement and connectivity. Thus our method provides a way for direct surface matching between any two ventricles.

For the surface of the cerebral cortex, our algorithm also provides a way to perform surface matching, while explicitly matching sulcal curves or other landmarks lying in the surface. Note that typically two surfaces can be matched by using a landmark-driven flow in their parameter spaces. An alternative approach is to supplement the critical graph with curved landmarks that can then

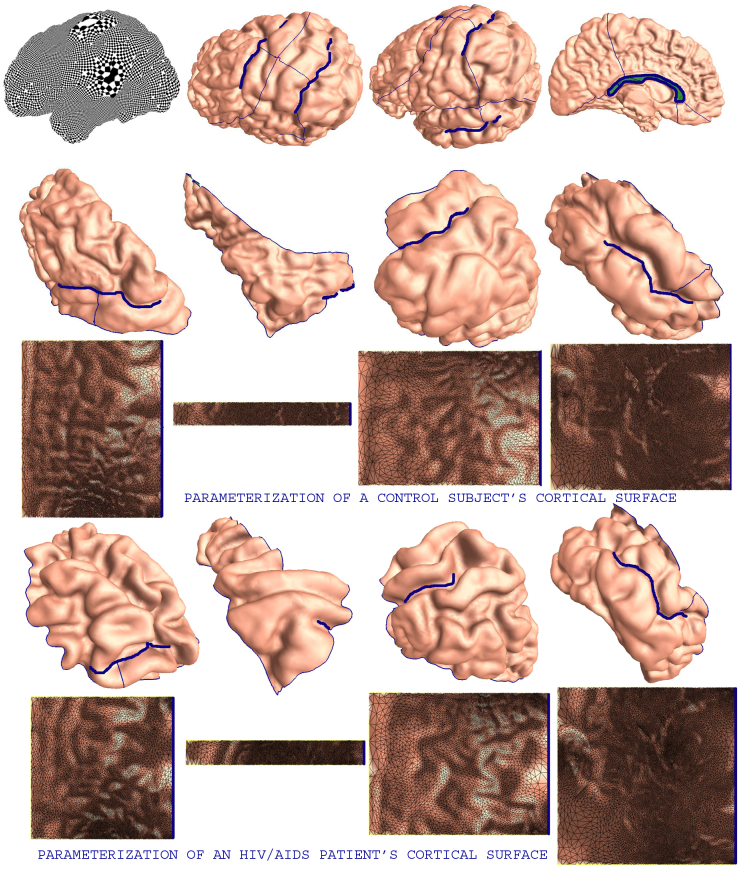

Fig. 2. Illustrates the parameterization of cortical surfaces using the holomorphic 1-form approach. The thick lines are landmark curves, including several major sulci lying in the cortical surface. These sulcal curves are always mapped to a boundary in the parameter space images.

be forced to lie on the boundaries of rectangles in the parameter space. This has the advantage that conformal grids are still available on both surfaces, as is a correspondence field between the two conformal grids. Figure 2 shows the results for the cortical surfaces of two left hemispheres. As shown in the first row, we selected four major landmark curves, for the purpose of illustrating the approach (thick lines show the precental and postcentral sulci, and the superior temporal sulcus, and the perimeter of the corpus callosum at the midsagittal plane). By cutting the surface along the landmark curves, we obtain a genus 3 open boundary surface. There are therefore two zero points (observable as a large white region and black region in the conformal grid; an illustration of the conformal structure is shown in the first panel the first row). We show cortical surfaces from two different subjects in Figure 2 (these are extracted using a deformable surface approach, but are subsequently reparameterized using holomorphic 1-forms). The second and fourth rows show the segmented patches for each cortical surface. The rectangles that these patches conformally map to are shown on the third and fifth row, respectively. Since the landmark curves lie on the boundaries of the surface patches, they can be forced to lie on an isoparameter curve and can be constrained to map to rectangle boundaries in the parameter domain. Although the two cortex surfaces are different, the selected sulcal curves are mapped to the rectangle boundaries in the parameter domain. This method therefore provides a way to warp between two anatomical surfaces while exactly matching an arbitrary number of landmark curves lying in the surfaces. This is applicable to tracking brain growth or degeneration in serial scans, and composite maps of the cortex can be made by invoking the consistent parameterizations. Lamecker et al's work [10] has the similar motivation as ours for the cortex case, which is to partition a surface into canonical patches and parameterize the patches with minimal distortion. However, our partition method is based on intrinsic Riemann surface structure and theirs is based on shortest paths along lines of high curvature. Thus our method is global and more stable.

4 Conclusion and Future Work

In this paper, we presented a brain surface parameterization method that invokes the Riemann surface structure to generate conformal grids on surfaces of arbitrary complexity (including branching topologies). We tested our algorithm on the hippocampus, lateral ventricle surfaces and on surface models of the cerebral cortex. The grid generation algorithm is intrinsic (i.e. does not depend on any initial choice of surface coordinates) and is stable, as shown by grids induced on ventricles of various shapes and sizes. Compared with other work conformally mapping brain surfaces to sphere, our work may introduce less distortion and may be especially convenient for other post-processing work such as surface registration and landmark matching. Our future work include automatic location of cutting positions and more experiments on disease assessment.

References

1. Thompson, P., et al. Nature **404** (2000) 190–193
2. Thompson, P., et al. In: Human Brain Mapping. Volume 9. (2000) 81–92
3. Schwartz, E., et al. IEEE PAMI **11** (1989) 1005–1008
4. Timsari, B., Leahy, R. In Hanson, K., ed.: Proc. SPIE, Medical Imaging. Volume 3979. (2000) 698–708
5. Hurdal, M.K., Stephenson, K. NeuroImage **23** (2004) S119–S128
6. Angenent, S., et al. MICCAI (1999) 271–278
7. Gu, X., et al. IEEE TMI **23** (2004) 949–958
8. Schoen, R., Yau, S.T.: Lectures on Harmonic Maps. International Press (1997)
9. Gu, X., Yau, S.T. Communication of Information and Systems **2** (2002) 121–146
10. Lamecker, H., et al. In: MICCAI, Springer (2002) 422–427