Brain Surface Conformal Mapping

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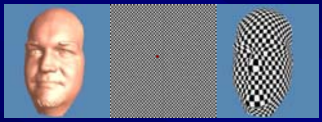
Abstract

We propose a new variational method which can find a unique mapping between any two genus zero manifolds by minimizing the harmonic energy of the map. We demonstrate the feasibility of our algorithm by applying it to the cortical surface matching problem. We use a mesh structure to represent the brain surface. Further constraints are added to ensure that the conformal map is unique. Empirical tests on MRI data show that the mappings preserve angular relationships, are stable in MRIs acquired at different times, and are robust to differences in data triangulation, and resolution. Compared with other brain surface conformal mapping algorithms, our algorithm is more stable and has good extensibility.

Conformal Mapping

- Any surface without holes or self-intersections can be mapped conformally onto the sphere
- This mapping, conformal equivalence, is one-to-one, onto, and angle preserving
- Locally, shape is preserved and distances and areas are only changed by a scaling factor
- A canonical space is useful for subsequent work

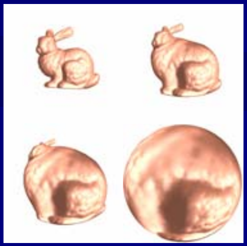


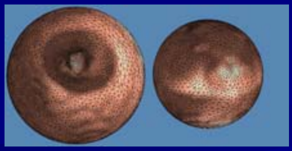
Conformal Mapping Properties

- Intrinsic to geometry
- Independent of triangulation and resolution
- Depends on metric continuously

Genus Zero Conformal Mapping Properties

- Harmonic is equivalent to conformal
- All conformal are equivalent
- All the conformal construct a automorphism group: Möbius group which is a linear rational group on complex plane and a 6 dimensional group.





Algorithm at a Glance

- Minimize Harmonic Energy
- Use absolute derivative
- All computation are on the target surface, without projecting to complex plane

Algorithm Deatails

- Harmonic energy $f: M \rightarrow S^2$

$$E(f) = \int_M \|\nabla f\|^2 d\sigma_M$$

- Discrete harmonic energy

$$E(f) = \sum_{[u,v]\in M} k_{uv}\|f(u) - f(v)\|^2 \qquad k_{uv} = \tfrac{1}{2}(\cot\alpha + \cot\beta)$$

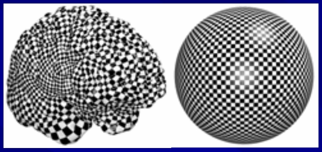
- Discrete Laplacian

$$\Delta f(u) = \sum_{[u,v]\in M} k_{uv}(f(u) - f(v))$$

Spherical parameterization algorithm for genus zero surface

- Use Gauss map as the initial degree one map
- Compute the gradient vector of harmonic energy on each vertex
- Project the gradient vector to the tangent space on S^2 at each vertex
- Update the image of each vertex along the tangential gradient direction
- Normalize the mapping by shifting the center of the mass to the sphere center

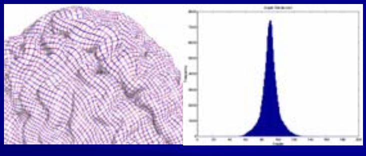
Experimental Results



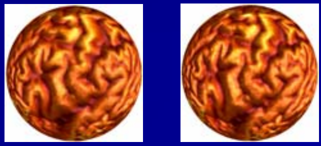
Conformal texture mapping. The conformality is visualized by texture mapping of a checkerboard image.

(a) (b) (c) (d)

Reconstructed brain meshes and their spherical harmonic mappings. (a) and (c) are the reconstructed surfaces for the same brain scanned at different times. Due to scanner noise and inaccuracy in the reconstruction algorithm, there are visible geometric differences. (b) and (d) are the spherical conformal mappings of (a) and (c) respectively; the normal information is preserved. By the shading information, the correspondence is illustrated.



Conformality measurement. The curves of iso-polar angle and iso-azimuthal angle are mapped to the brain, and the intersection angles are measured on the brain. The histogram is illustrated.



(a) (b)

Conformal mappings of surfaces with different resolutions. The original brain surface has 50,000 faces, and is conformally mapped to a sphere, as shown in (a). Then the brain surface is simplified to 20,000 faces, and its spherical conformal mapping is shown in (b).

Optimize the Conformal Parameterization by Landmarks

- We define a metric to measure the quality of the parameterization.
- Suppose two brain surfaces S^1,S^2, two conformal parameterizations are denoted as f1: $S^2 \rightarrow S^1$ and f2: $S^2 \rightarrow S^2$, the matching metric is defined as

$$E(f_1, f_2) = \int_{S^2} \|f_1(u,v) - f_2(u,v)\|^2 du dv$$

- Let Ω be the group of Möbius transformations. We can compose a Möbius transformation such that

$$E(f_1, f_2 \circ \tau) = \min_{\zeta\in\Omega} E(f_1, f_2 \circ \zeta)$$

- Landmarks are commonly used in brain mapping. They are a set of sulcal curves manually drawn on the brain surfaces.
- We can use landmarks to obtain such a Möbius transformation.

the Conformal Parameterization by Landmarks (Cont.)

- Landmarks are represented as discrete point sets. We can reduce the brain matching metric by reducing the matching metric on landmark sets.
- First we project the sphere onto the complex plane. We find a Möbius transformation on the complex plane which reduce the matching metric on landmark sets. Then we project the results back to the sphere.
- For a Möbius transformation on the complex plane u, since it maps infinity to infinity, it means the north poles of the spheres are mapped to each other.
- Then u can be represented as a linear form az+b. Let p_i and q_i, i=1 …n, are corresponding landmark points. The functional of u can be simplified as

$$E(u) = \sum_{i=1}^{n} g(z_i)|az_i + b - \tau_i|^2$$

- where z_i is the stereo-projection of p_i, τ_i is the projection of q_i, g is the conformal factor from the plane to the sphere.

Landmark Experimental Results

Subject	Vertex #	Face #	Before	After
A	65,538	131,072	-	-
B	65,538	131,072	604.134	506.665
C	65,538	131,072	414.803	365.325

More Genus Zero Surface Examples

Discussion

- Compared with Haker's method [I] , our method is more geometric; no big distortion areas; more stable; good extension ability (e.g. it is possible to do brain mapping between two brains using our algorithm.)
- Compared with Hurdal's method [II] , our method preserves angles; good mapping between brains and the canonical space.

Brief Reference

I. *S. Haker, S. Angenent, A. Tannenbaum, R. Kikinis, g. Sapiro, and M. Halle*. "Conformal Surface Parameterization for Texture Mapping". IEEE Transactions on Visualization and Computer Graphics, 6(2):181-189, April-June 2000

II. *M.K. Hurdal, K. Stephenson, P.L. Bowers, D.W.L. Summers, and D.A. Rottenberg.* Coordinate Systems For Conformal Cerebellar Flat Maps. In NeuroImage, Vol. 11: S467, 2000