The Snowblower Problem

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Suppose that your backyard looks like this:

One morning you wake up and it is covered with snow…

…uniformly covered

So you pull out your **snowblower**…

…or your **Snowblower**

...or your **SNOWBLOWER**

…and begin **snowblowing**

Depending on your backyard, snowblowing may look like this…

or snowblowing may look like this:

This talk: Algorithmic Aspects of Snowblowing

- Introduce the **snowblowing problem** (intermediate between TSP and material-handling problems)
- Give O(1) approximation algorithms for several versions of the problem
- Prove NP-hardness for some versions of the problem

Snowblower: Material-Shifting Machine

- It **lifts** snow from one location, and **piles** it on an adjacent location

We must respect max snow depth (height) *D*, because…

…if we pile snow up too high…

…the snowblower gets **stuck**

Where to **dispose of the snow**?

Where to **dispose of the snow**?

(neighbor's yard)

We can pile snow arbitrarily high on the neighbor's lawn…

Effectively the neighbor's lawn has **infinite capacity**

Alternatively: boundary is "**cliff**"
We dump as much snow as we want

(A bigger cliff)

We can achieve infinite capacity using a **snow melter**…

Snow Melter

SNOW MELTER

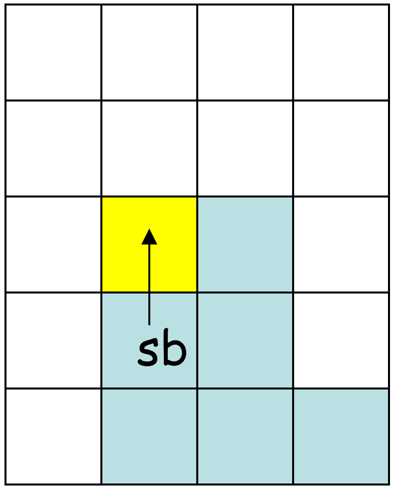
SB Problem Definition - Driveway

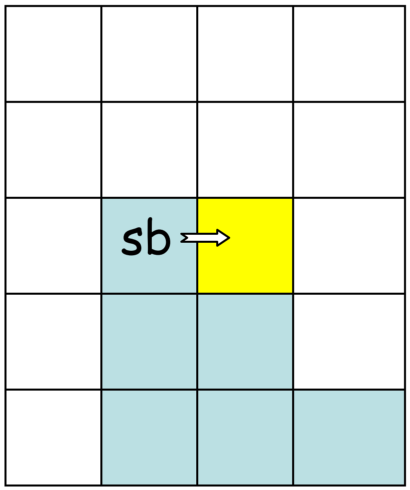
- Polygonal domain P
  - integral-orthogonal and pixelated
  - no holes
- SB

  1 pixel (initially in garage)

SB Problem Definition

- SB moves from pixel to adjacent pixel
  - picks up all snow
  - throws onto a neighbor pixel
  - or over the boundary of region
  - max depth of snow $D \geq 2$
- **Objective: minimize the length of the path of the snowblower**

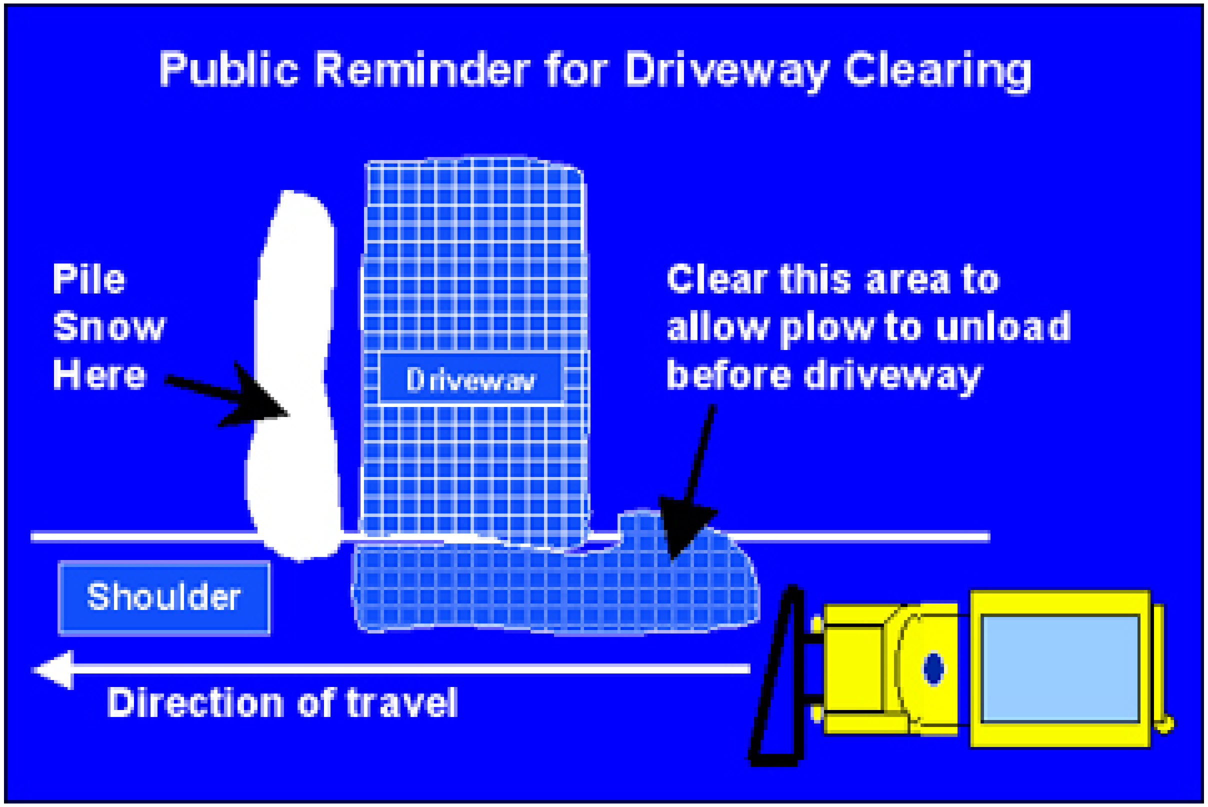




The SBP is TSP-like

- Milling/lawnmowing [Arkin,Fekete,Mitchell00, ArkinHeldSmith00, Held91]
  - visit = remove
    - never re-visit ⇨ in NP
- Material handling [pushing blocks; extensive OR literature]
  - visit = move
    - may need to re-visit a lot ⇨ in NP?
- The Snowblower Problem *(SBP)*
  - visit = move
    - stacking ≤ D allowed
  - visit a boundary pixel = remove
    - our algs ⇨ in NP

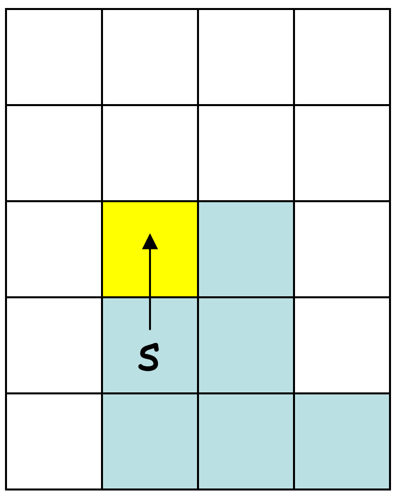
We are not the first to consider pixel environments for snowblowing…



City of Danville
Public Works Department
Danville, VA 24540

Throw Direction

On which pixel can snow be placed?



Right

Left

Forward?

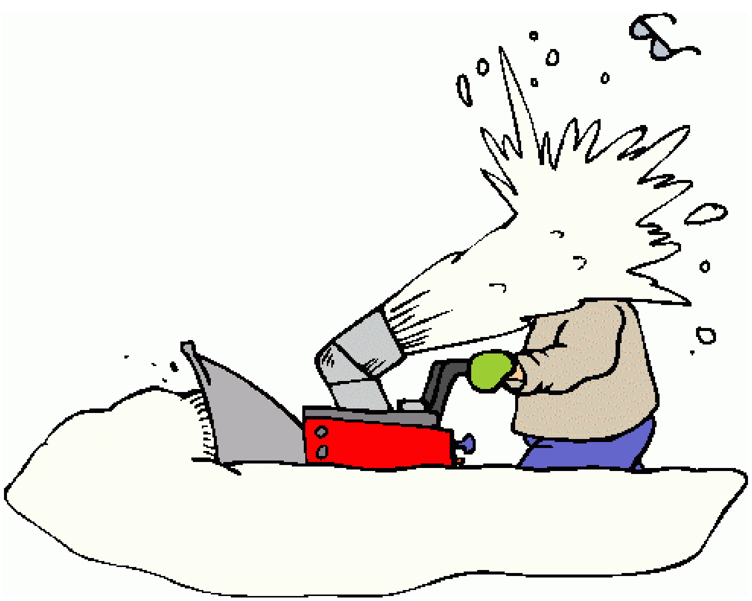
Yes

- if adjustable

Backward?

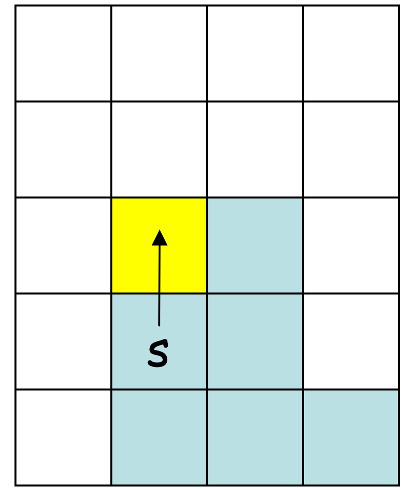
Backward?

- Not easy to implement. But it makes the algorithms easier.

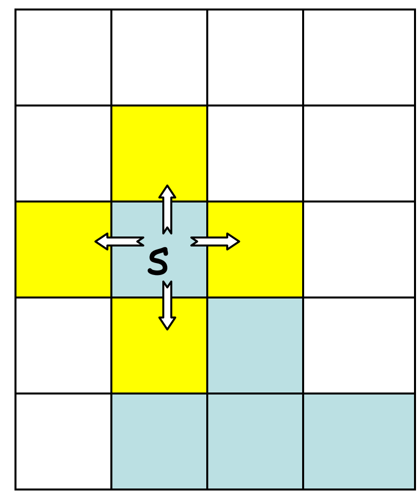


Default Model

- Throwback allowed
- Not intended to be realistic, but easy to describe and other models reduce to it



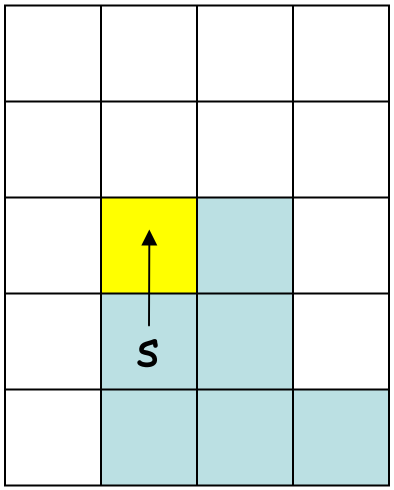
8-APX

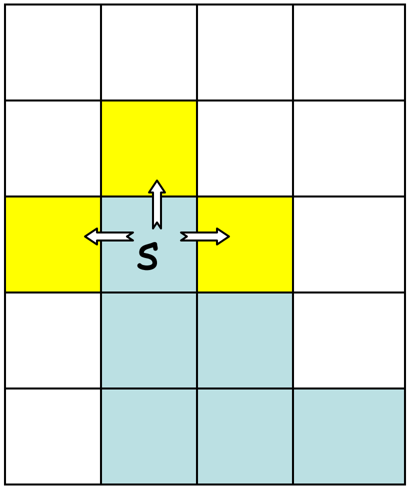


Adjustable-Throw Model

Right, left, or fwd

9-APX

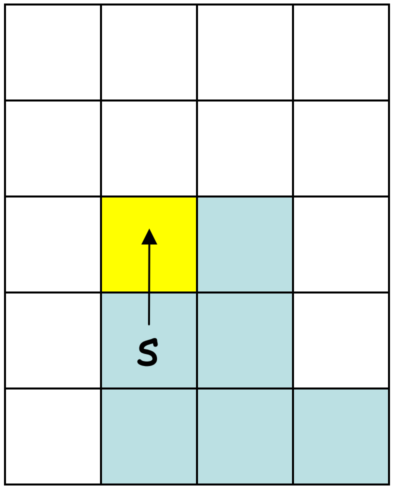


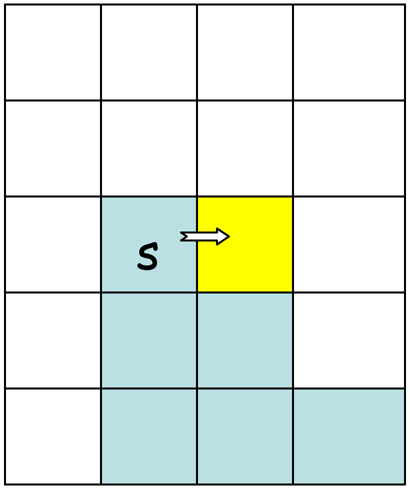


Fixed-Throw Model

Right only

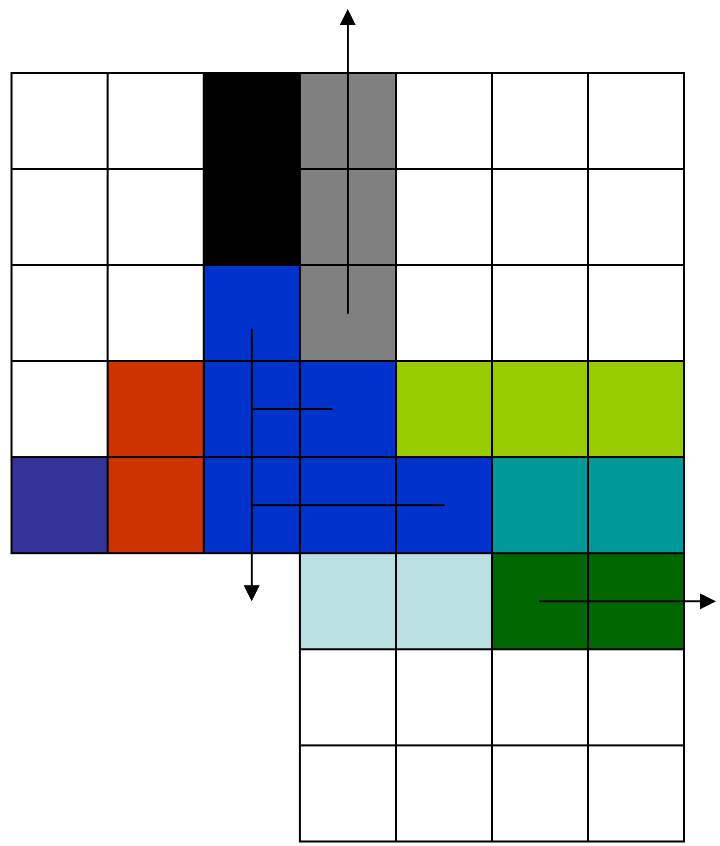
106-APX





A Key Idea

- Voronoi decomposition
  - closest bndry pixel edge
    - tie-breaking
  - clear Voronoi-cell-by-Voronoi-cell
- Lower Bounds
  - snow amount
  - distance to boundary



Lower Bounds

- snow LB

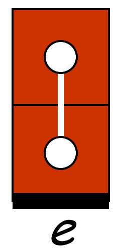
  snowLB(R) = # of pixels of region *R* with snow

- distance LB

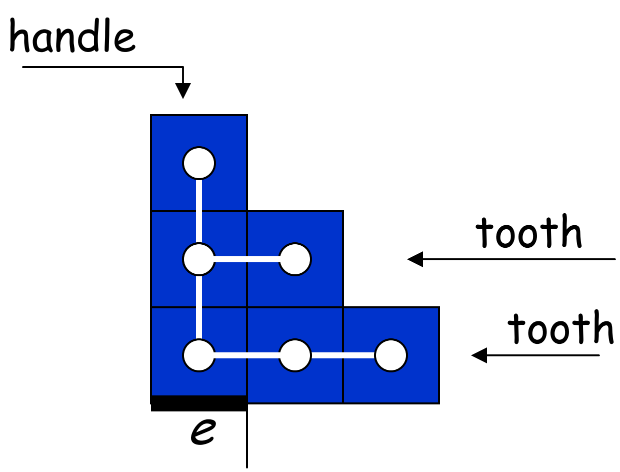
  $$distLB(R) = \frac{1}{D} \sum_{\text{pixel} \in R} [\text{distance from pixel to the boundary}]$$

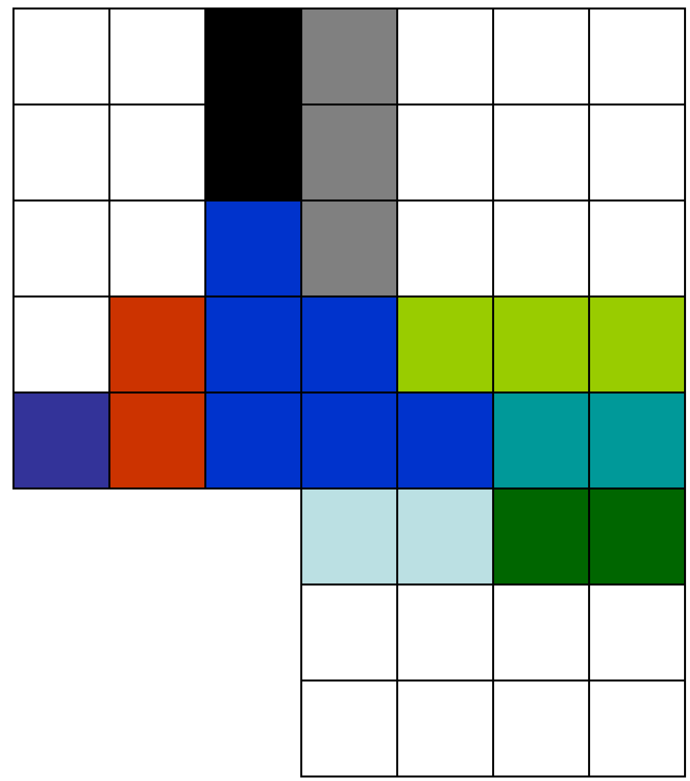
Boundary Cells Are Of Two Types (by our tiebreaking rule)

- Lines



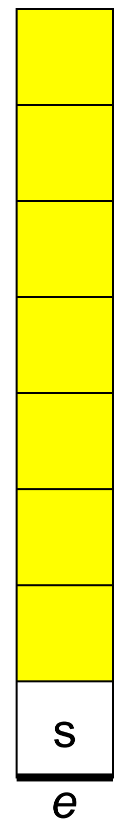
- Combs





Line-clearing (D=3)

- Move up D, doing back throws
- U-turn
- Forward throw moving down to boundary



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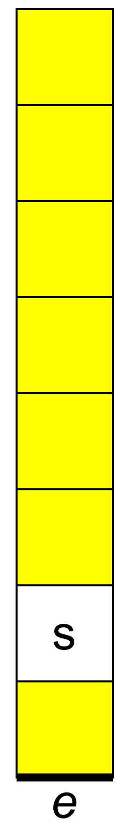

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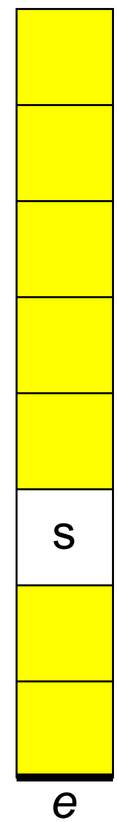

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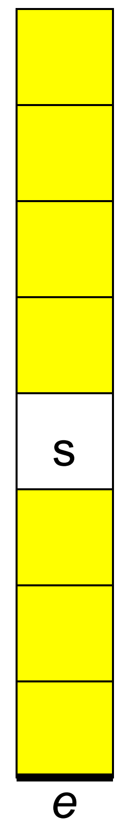

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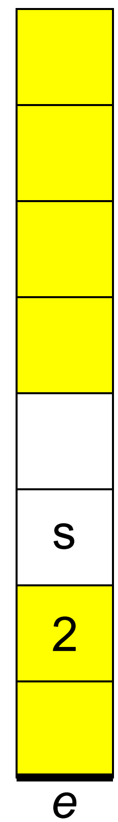

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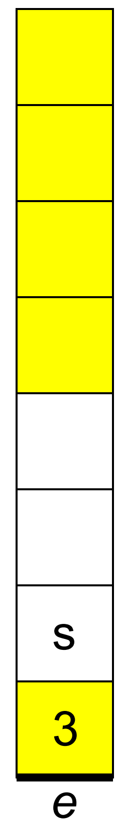

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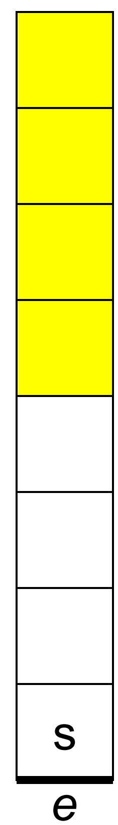


Cost of Line L

- Each D-full pass (clearing D snow units)

  $distLB(cleared) = (h+1+\ldots+h+D)/D$

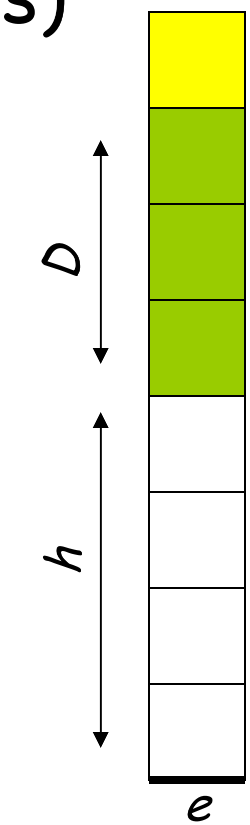
  $\sim h+D/2$

  $cost = 2(h+D)$

  $\leq 4\ distLB(cleared)$

- The pass that is not D-full

  $cost \leq 2\ snowLB(L)$

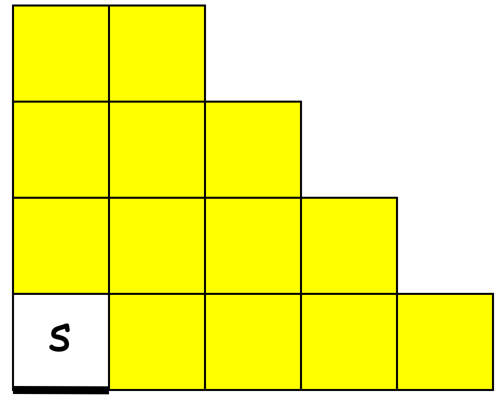


- Total cost to clear line L

  $cost(L) \leq 4\ distLB(L) + 2\ snowLB(L)$

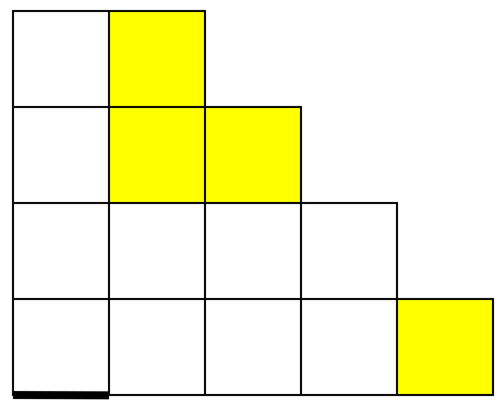
Clearing a Comb (*D* = 3)

- First clear whatever lines we can using *D*-full passes (where clear *D* units of snow)



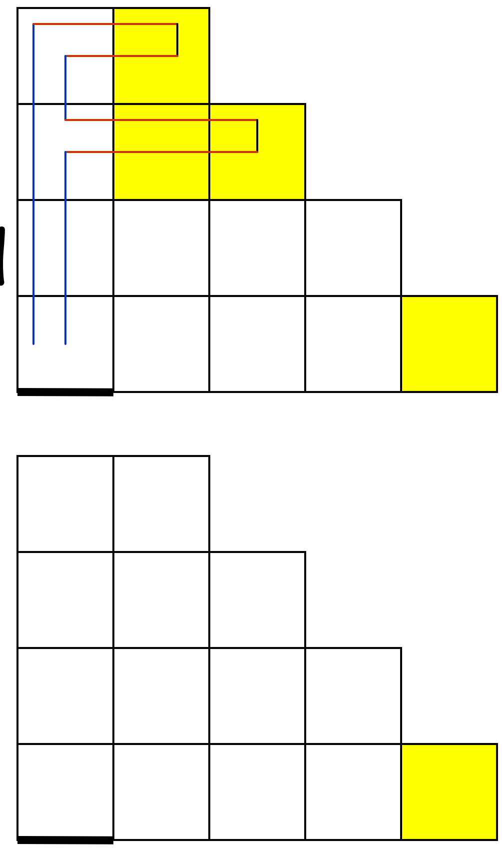
- *cost(L) ≤ 4 distLB(L)*

- Now the comb is ready for another operation: "brush-ready".



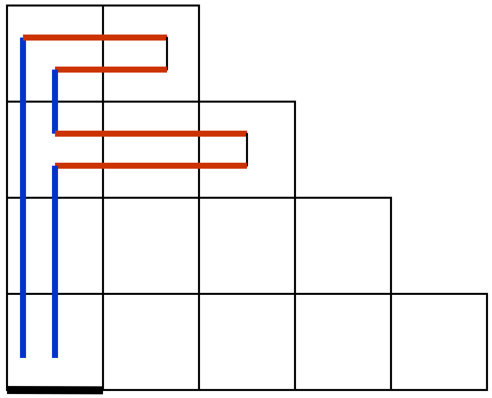
Brush Operation for Clearing Combs

- "Capacitated DFS"
  - Proceed tooth by tooth
  - until *D* units of snow moved
    - clear a tooth and move down



Cost of a Brush (red part)

- A brush
  - through handle
  - through teeth

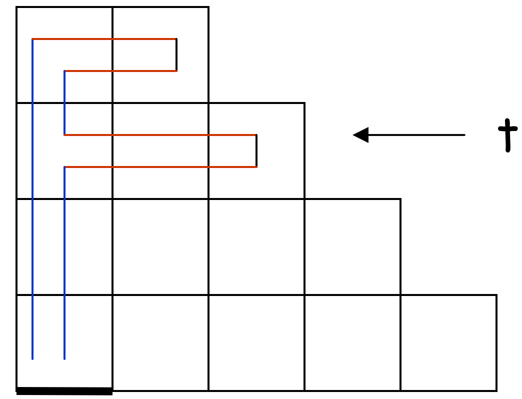


- Each tooth is visited ≤ 2 times (there & back twice)
  [Because brush-ready we know *snow(tooth) < D*]

- For all brushes
  cost(red) ≤ 4 snowLB(teeth)

Cost of a Brush (blue)

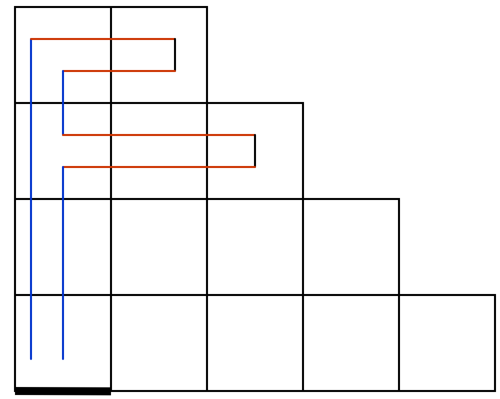
- We charge the cost of the blue part of the brush to the distLB of the snow cleared in previous brush.



- $cost(blue) \leq 2\ distLB(cleared) + 4\ snowLB(handle)$

All Brushes

- cost(brushes) =
  cost(blue) + cost(red)
  ≤ 4 snowLB(comb) +
  2 distLB(cleared)

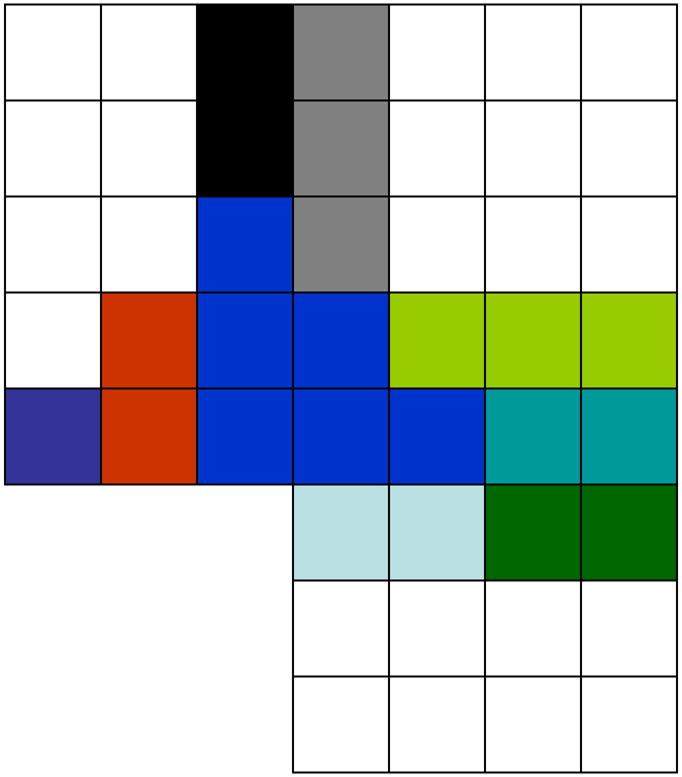


Comb Clearing

- cost(comb) =
  cost(line-clearing) + cost(brushes)
  ≤ 4 snowLB(comb) + 4 distLB(comb)

Cost of Polygon *P*

Treat each vornoi cell independently…

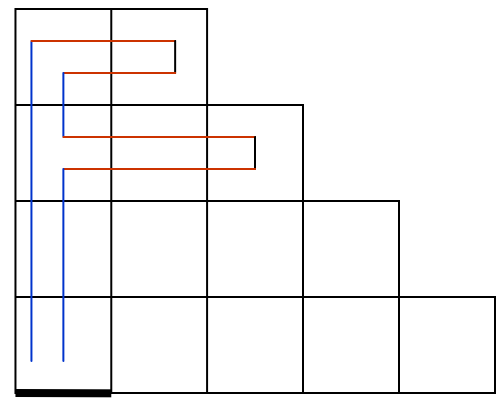


- $cost(L) \leq 4\ snowLB(L) +$
  $2\ distLB(L)$
- $cost(comb) \leq 4\ snowLB(comb) +$
  $4\ distLB(comb)$
- $cost(P) \leq 4\ snowLB(P) +$
  $4\ distLB(P)$
- $OPT \geq snowLB(P), distLB(P)$

8-approx

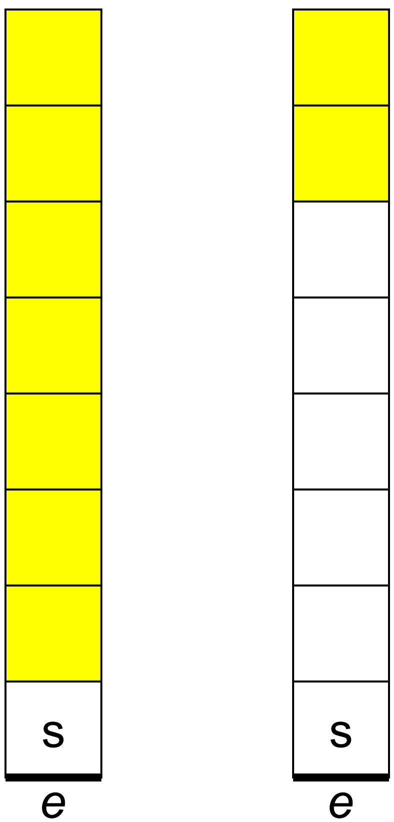
Other Throw Models

- Idea: simulate backthrow and reduce to the default model
- One issue: snow doesn't travel directly to its Voronoi edge

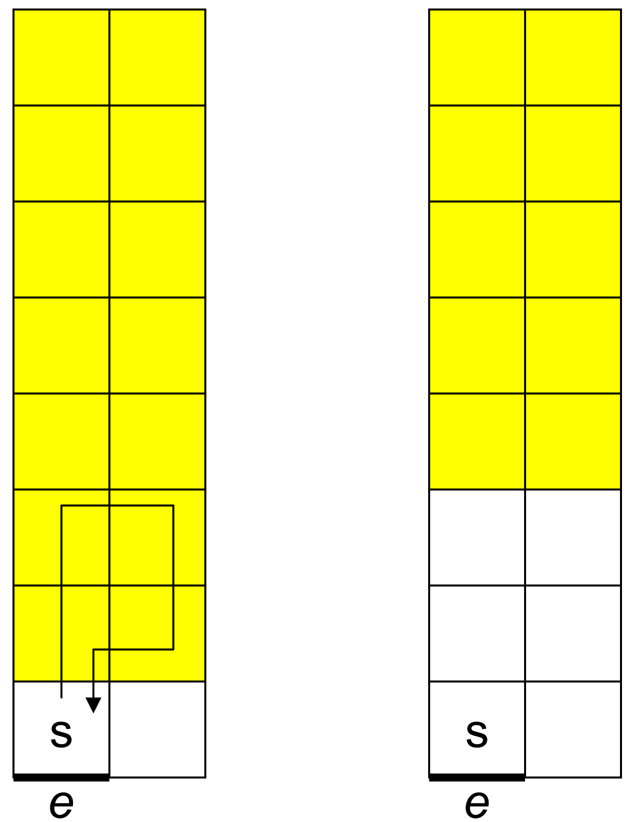


Line-clearing

- D-full pass

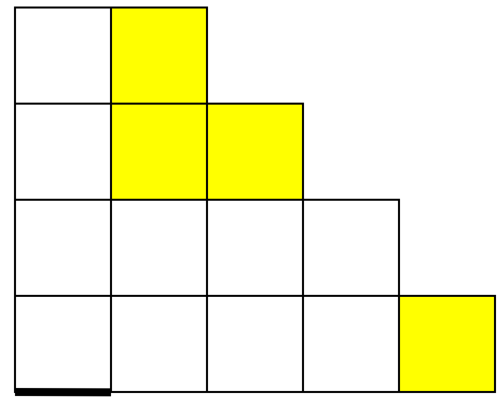


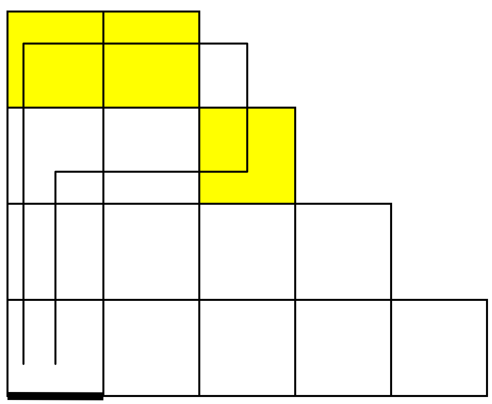
- $\lfloor D/2 \rfloor$-full pass

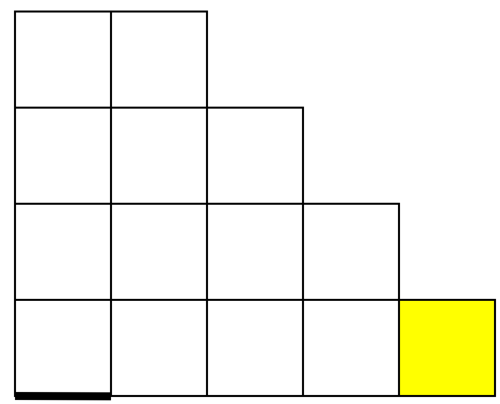


Brush

- $D \longrightarrow \lfloor D/2 \rfloor$
  with any cleared pixel
  ≤ 1 "extra" pixel is "touched"
  brush is feasible







Adjustable Throw

$(4 + 3D/\lfloor D/2 \rfloor)$-*approx*

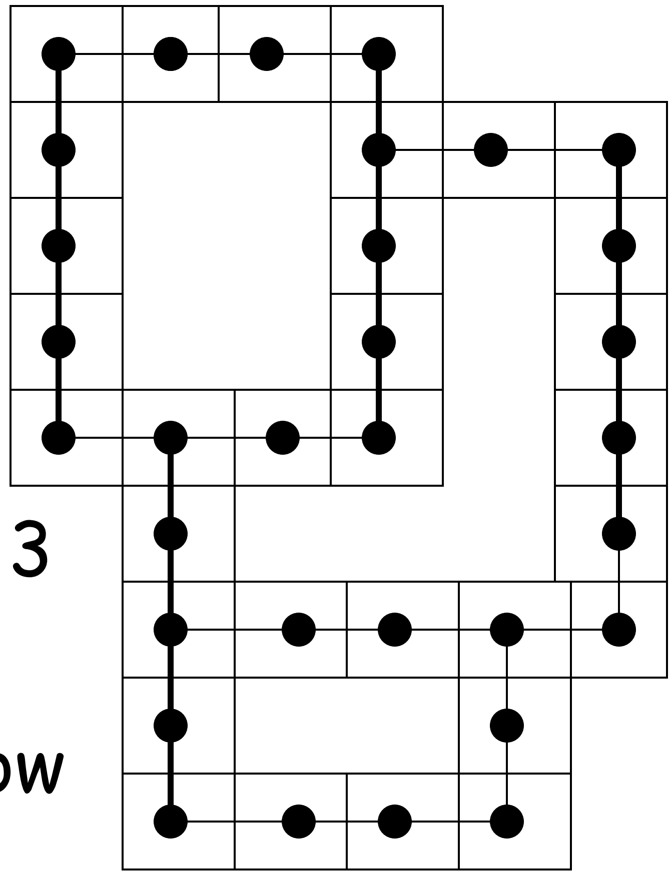
Fixed Throw

$(34 + 24D/\lfloor D/2 \rfloor)$-*approx*

more involved emulations

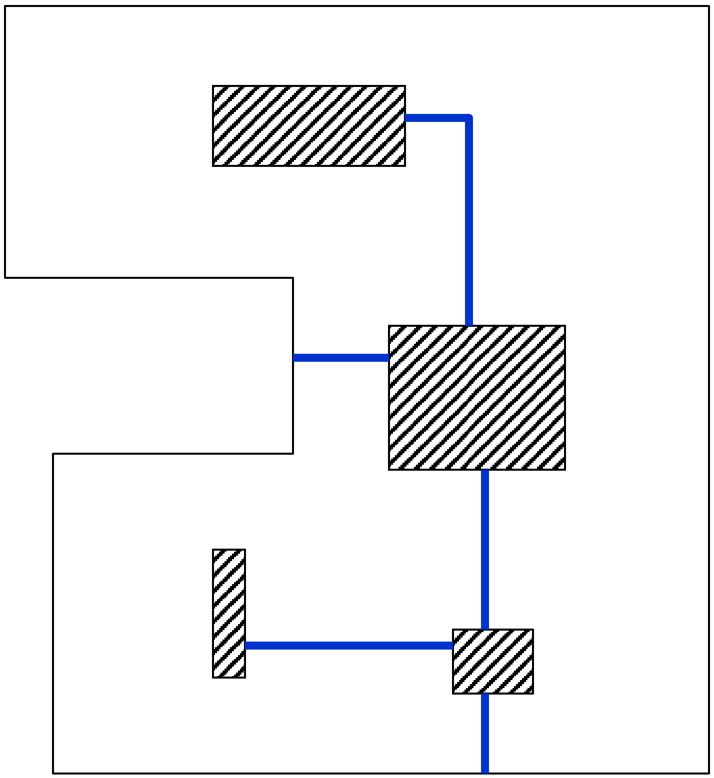
Hardness

- In NP
  - by our algorithms
- NP-hard
  - Hamilton Cycle in deg ≤ 3 grid graphs
  - default/adjustable throw
  - holes



Polygons with holes

- Holes as obstacles
  - verbatim
- Holes as cliffs
  - bridge holes
    - width-2 paths
  - same alg
    - bd pixels around the tree
  - increases approx factor



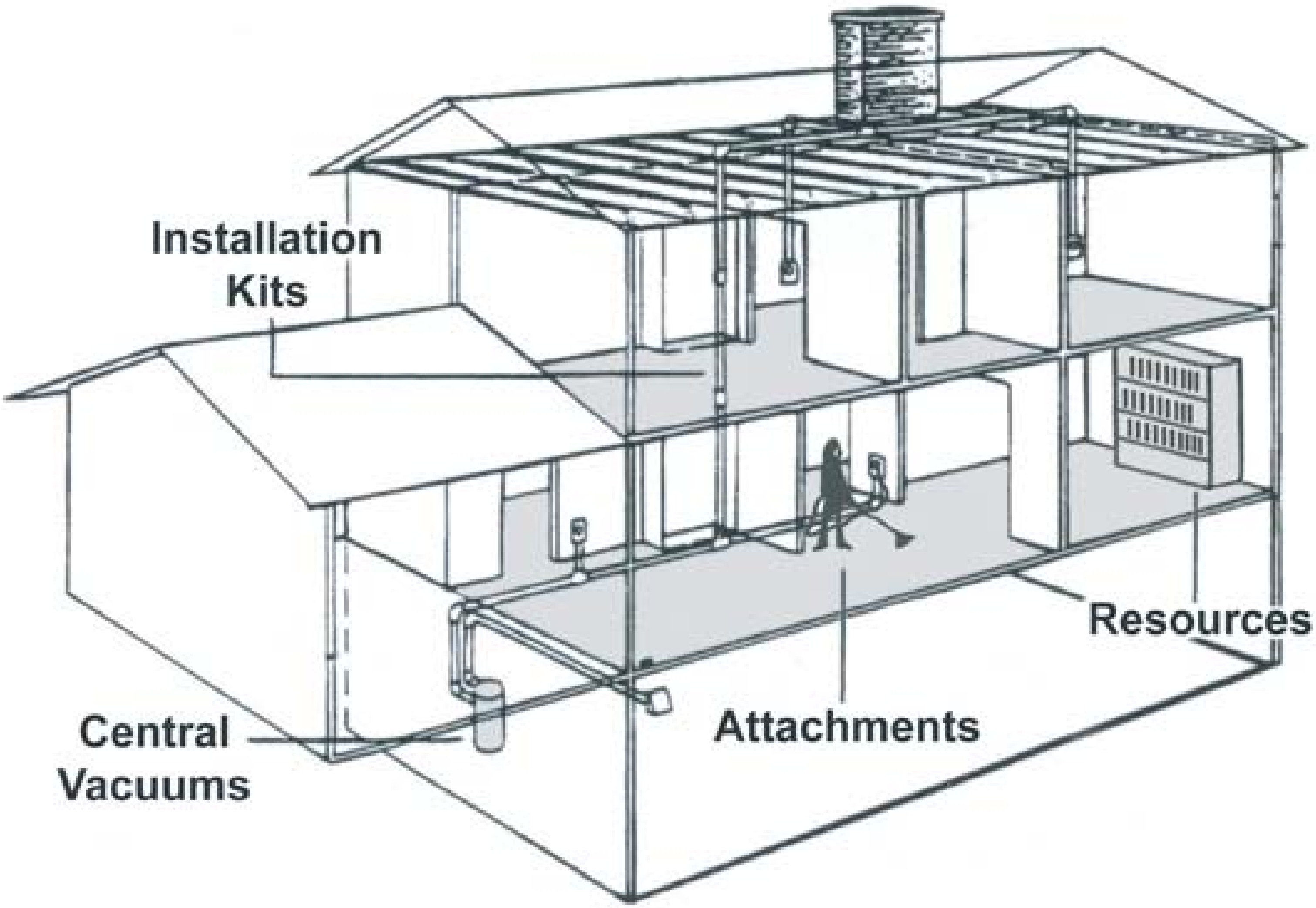
Nonuniform Initial Depth

- Straightforward generalization
  - approx factors depend on *D* linearly

Nonrectiliear

- Once around boundary
  - apply algorithms

Central Vacuum System



Dustpan Vac in Baseboard

"Infinite" Capacity

Vacuum Cleaner Problem

Robot

Exactly the SBP
default model

Conclusion

SBP

3 throw models

- default
  - vacuum-cleaner
- adjustable
- fixed

O(1)-approx (8,9,106)

NP-complete

- default/adjustable
- holes

Open

- Hardness
  - simple polygon
  - fixed throw

- Improve approx factors

Turn Cost

Online

Throwing >1 away

Multiple SBs