Asynchronous Shared-Memory Mutual Exclusion in $O(\log^2 \log n)$ RMRs

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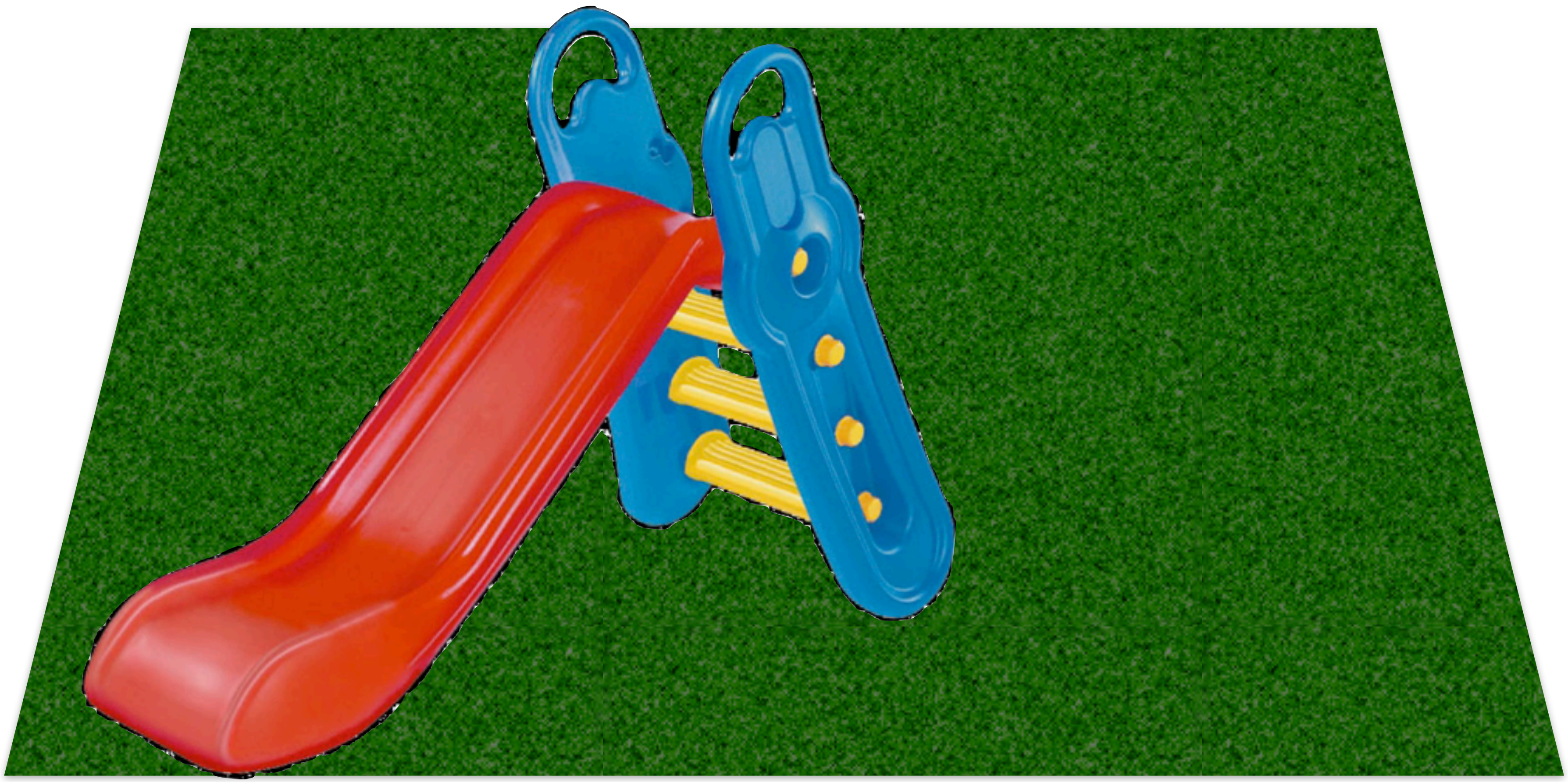
How to share… efficiently



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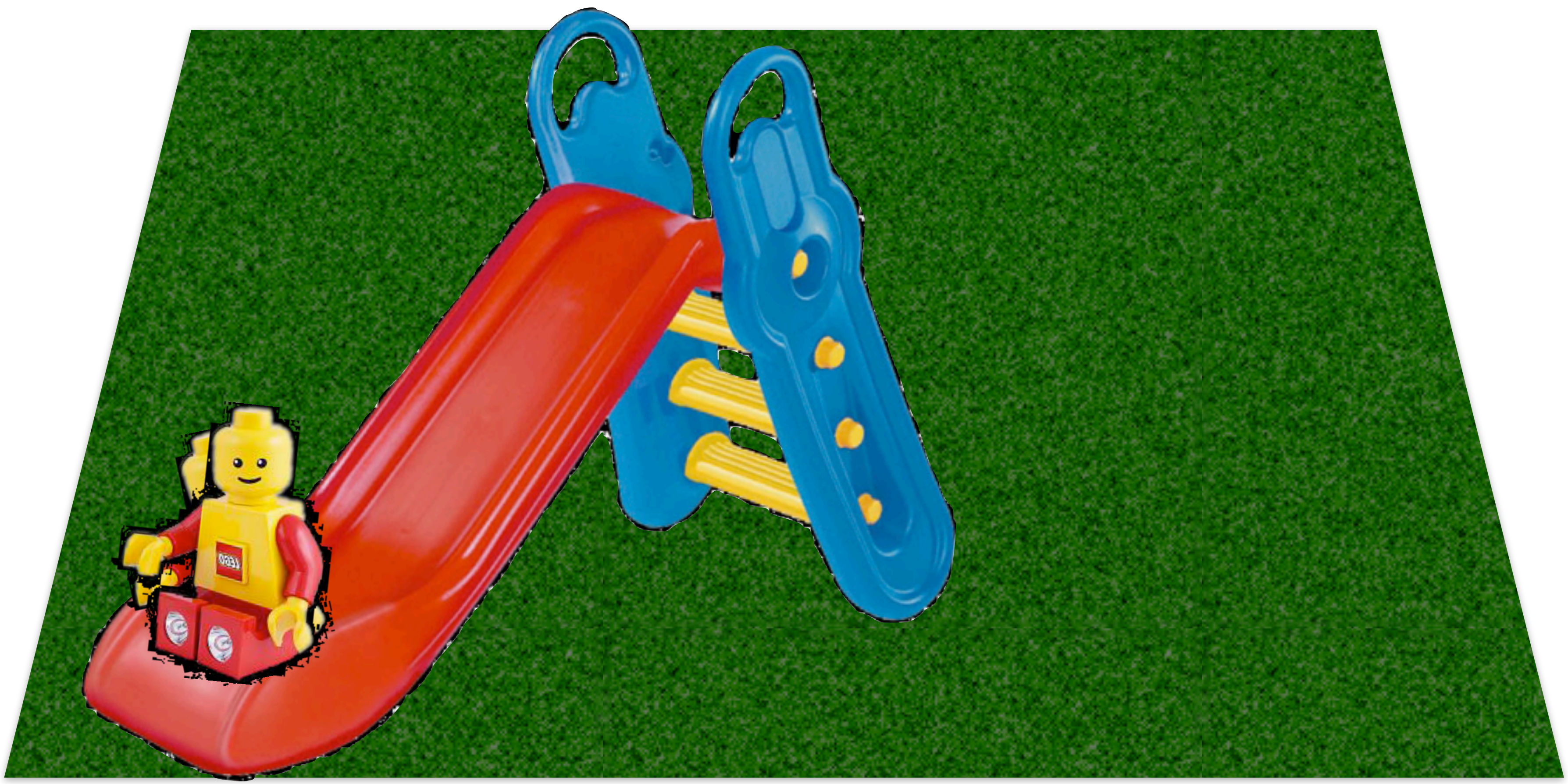

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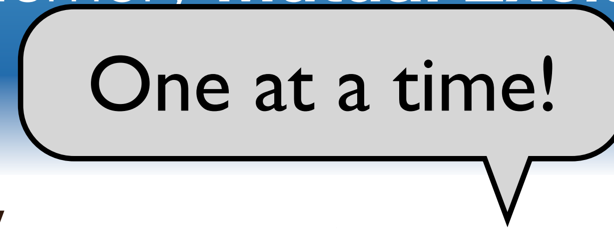

How to share… efficiently

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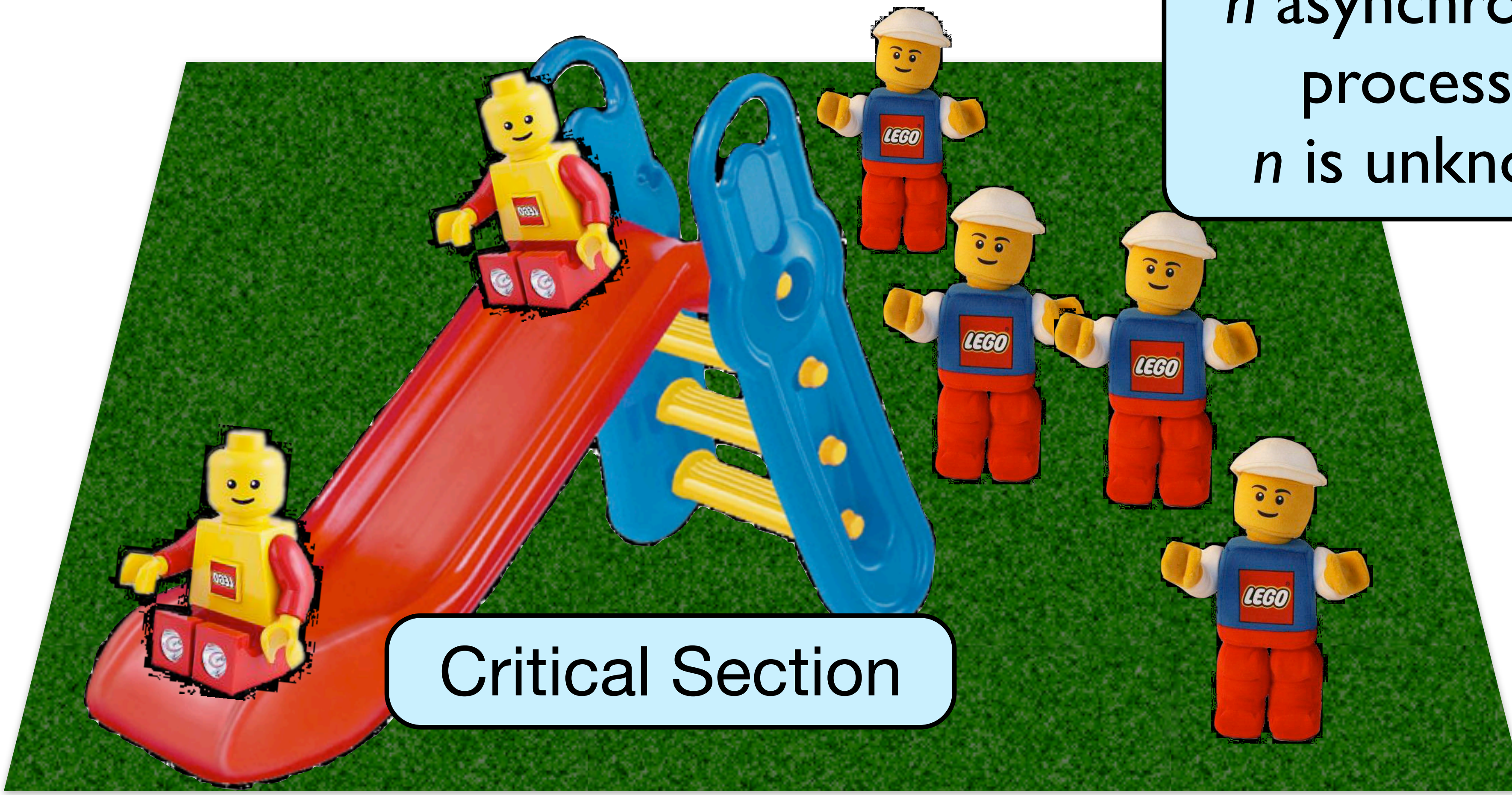
Asynchronous Shared-Memory **Mutual Exclusion** in $O(\log^2 \log n)$ RMRs

n asynchronous processes.
n is unknown.



Objective: each process should pass through the critical section exactly once.

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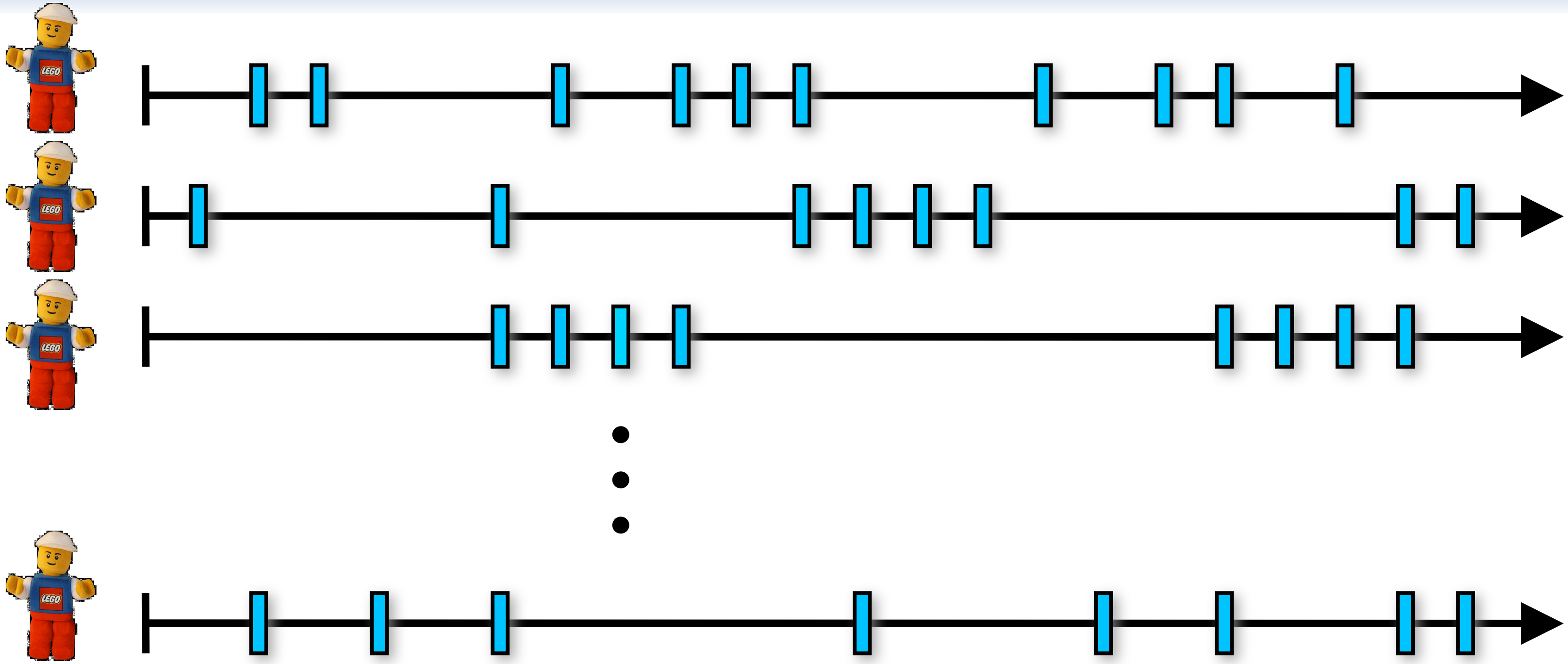

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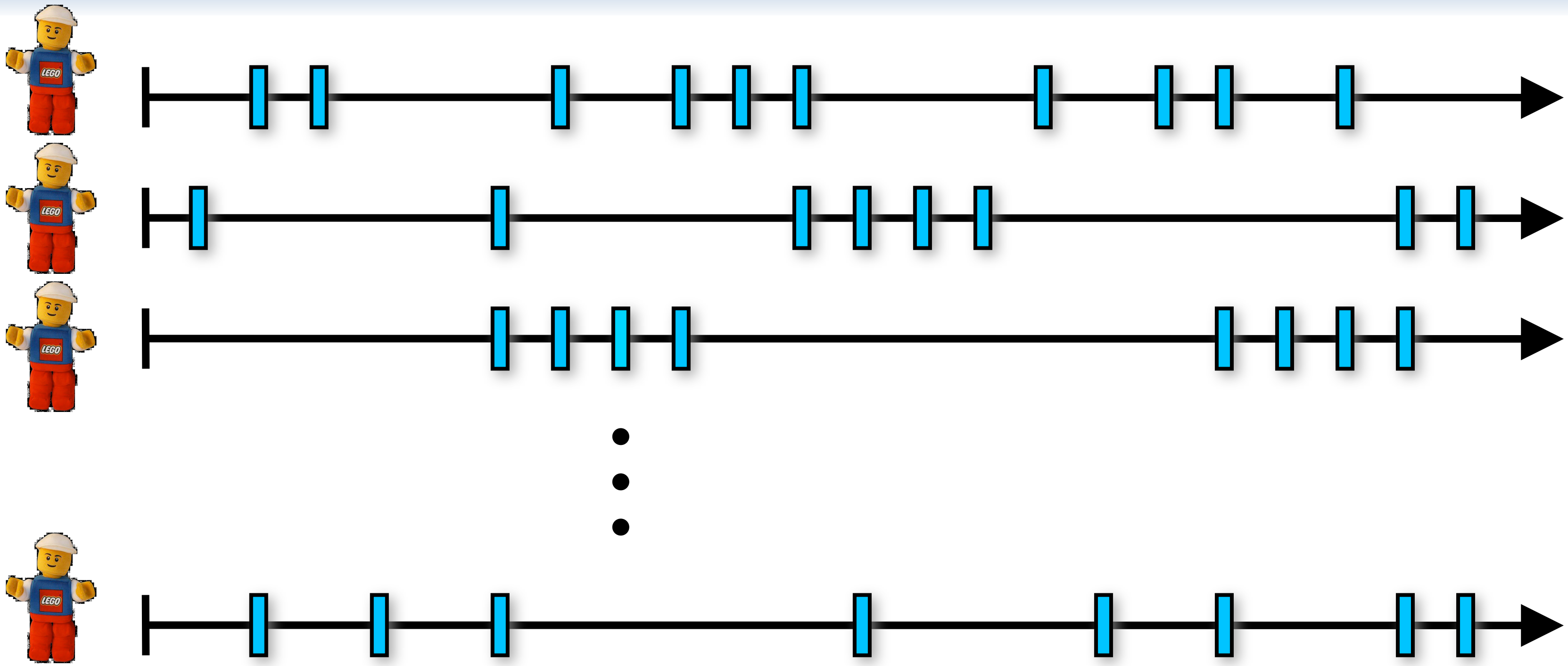
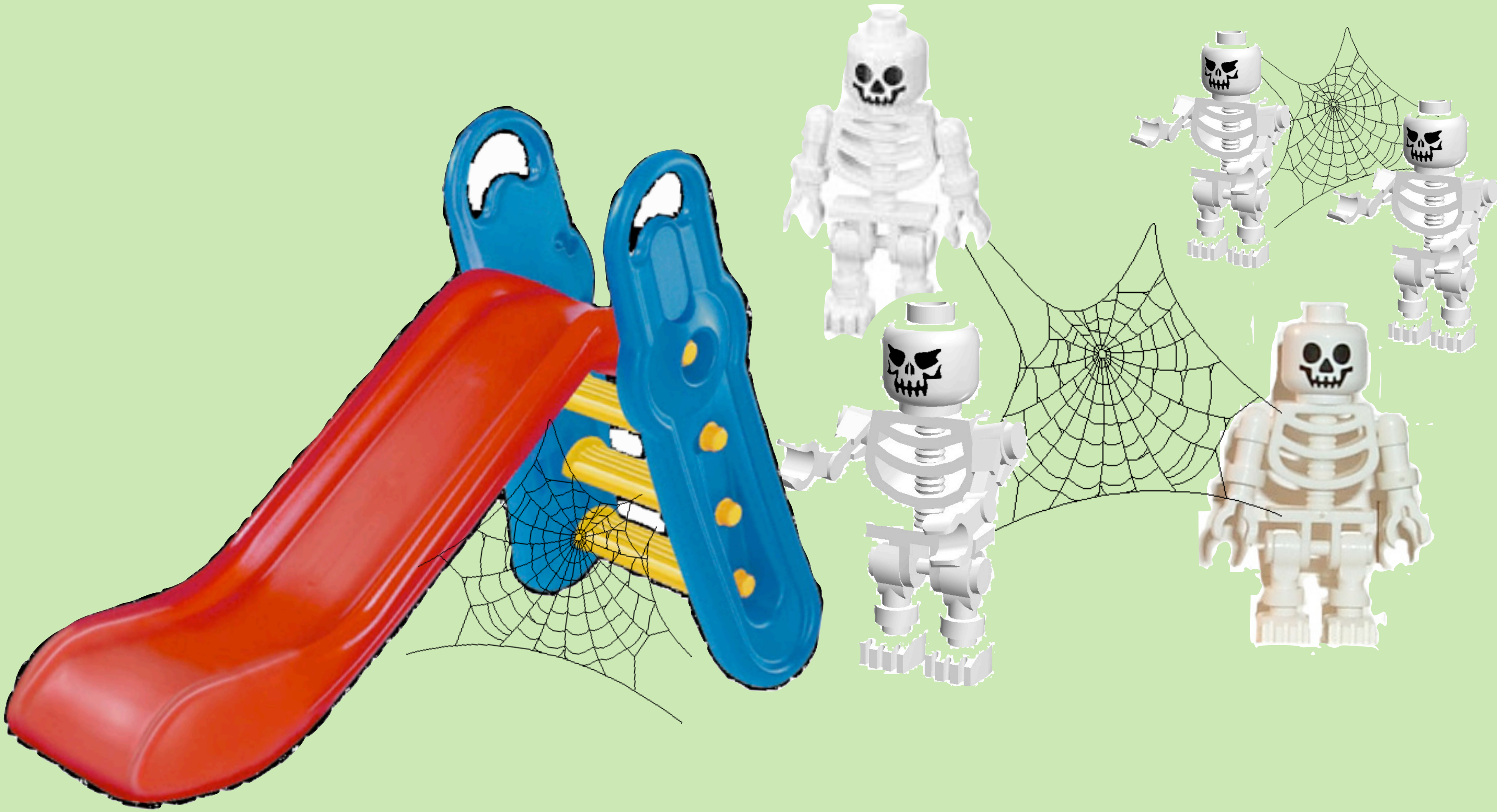


An “adversary” determines schedule.

- **Full-knowledge**: knows all but future coin flips.
- **Oblivious**: Determines schedule in advance.

We don't want deadlock.

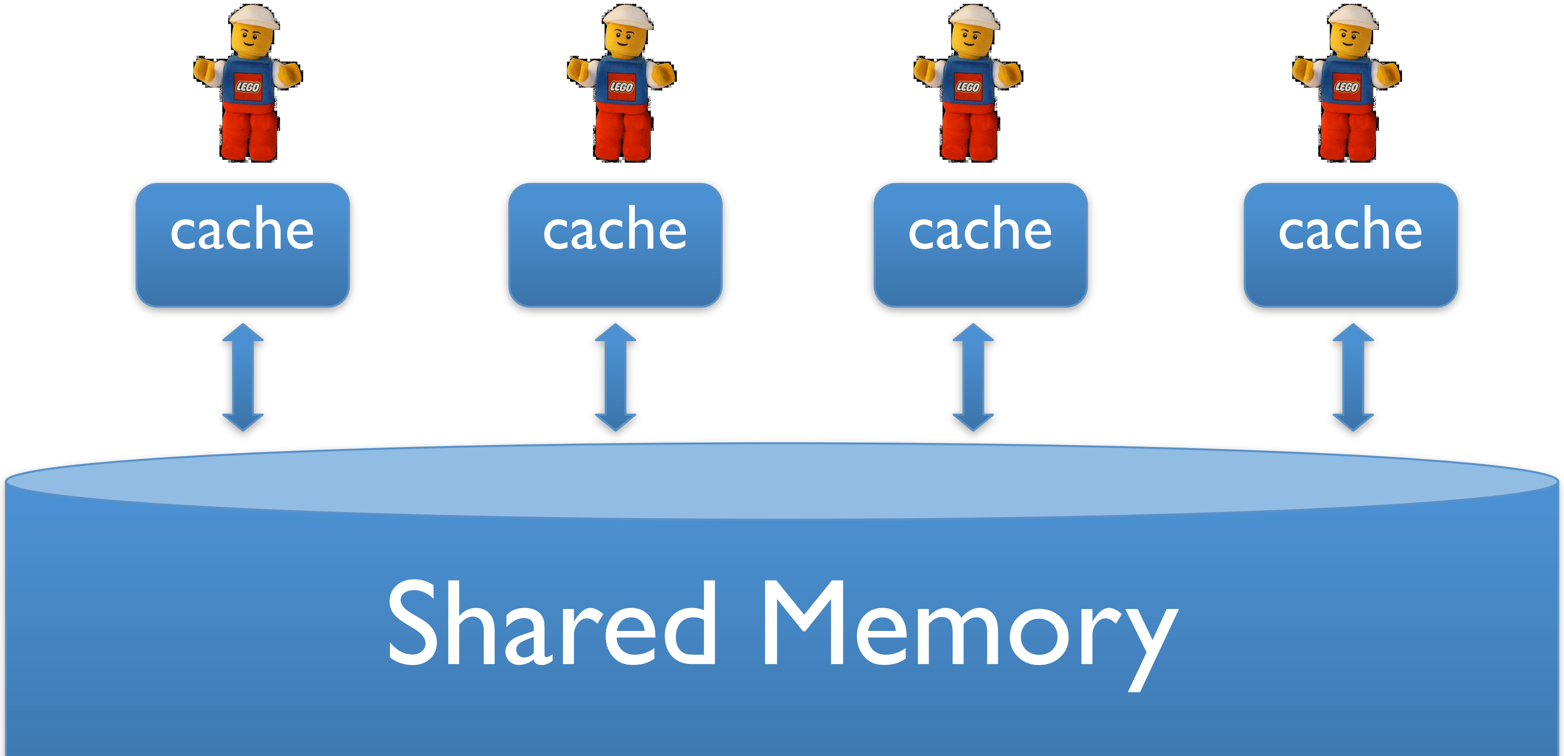


Cache-coherent shared memory:

- System keeps caches consistent.

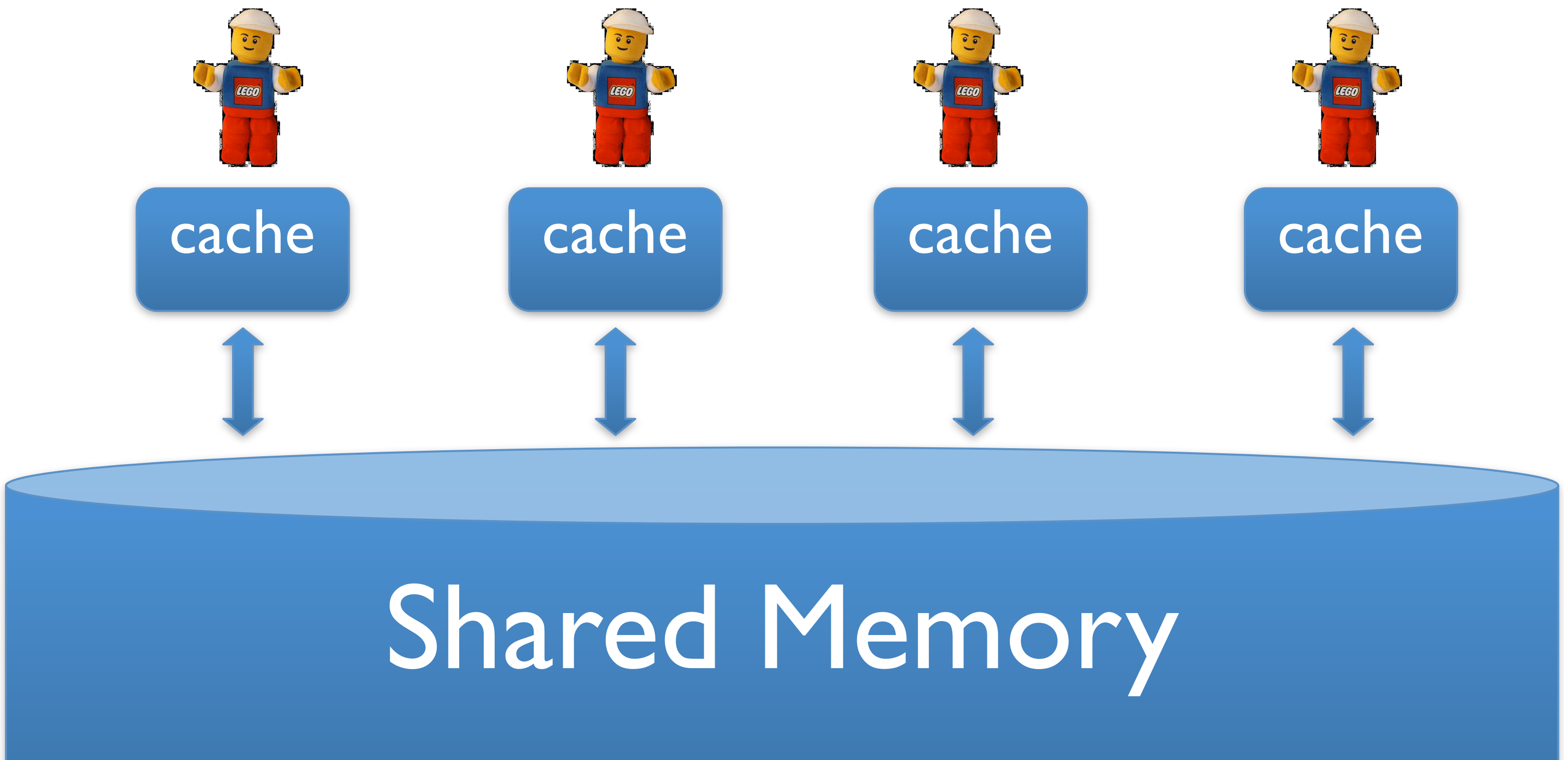
Cost model:

- Accessing local cache is effectively free.
- Accessing shared memory is expensive.



Asynchronous **Shared-Memory** Mutual Exclusion in $O(\log^2 \log n)$ **RMRs**

- Remote memory reference (atomic read/write): O(1).
- Spinning on a local variable is free,
  but costs 1 each time the variable changes.
- Compare and swap (CAS): O(1).
  [Golab et al.]. Uses atomic reads/writes + spinning.



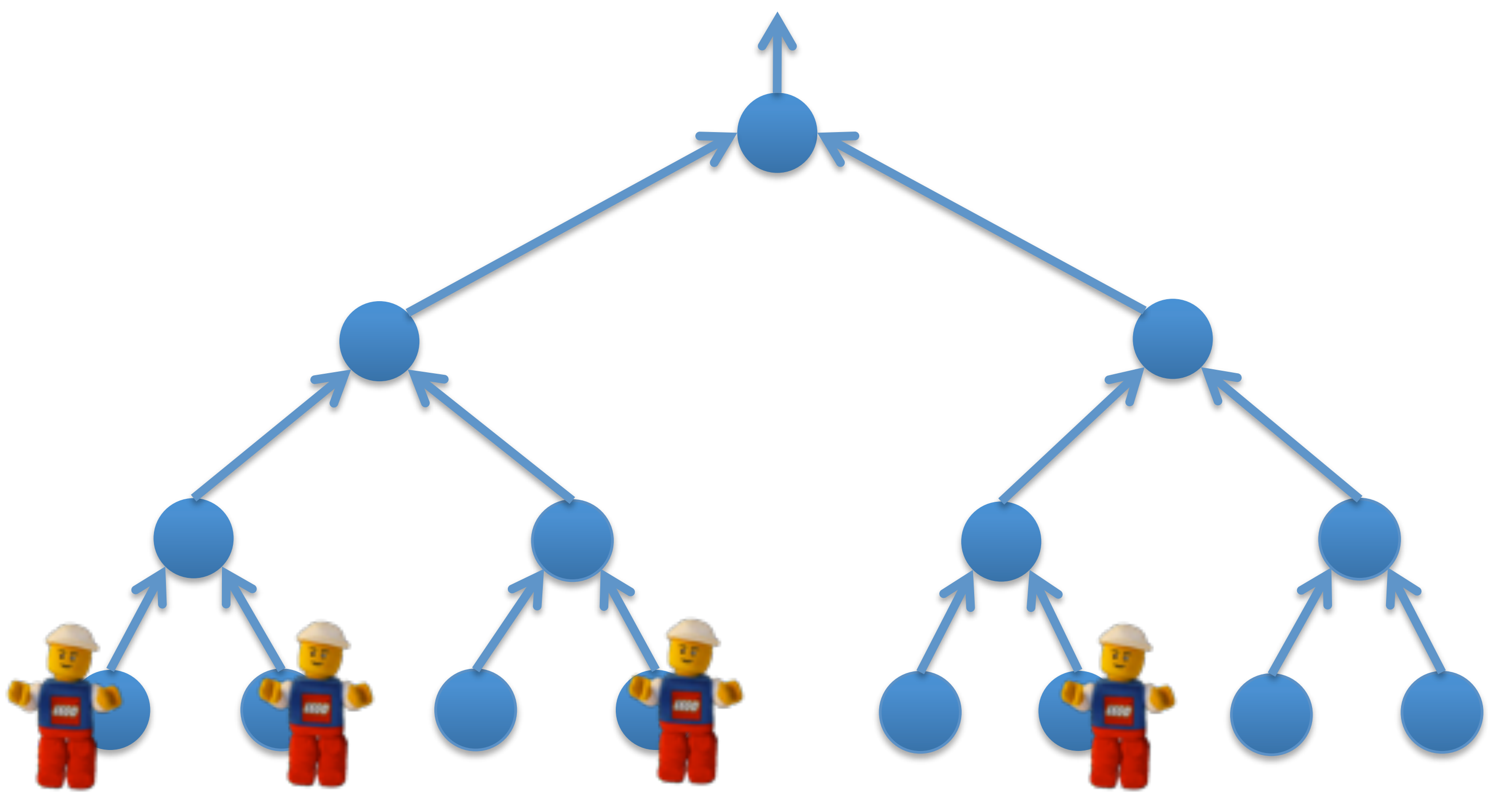
Naive Solution: O(*n*) RMRs per process.

- Protect the critical section with a lock

- Upon arrival: try to acquire (CAS) lock.
- Repeat:
  - Spin on lock until it becomes available.
  - Try to acquire (CAS) lock.

Better solution: O(log *n*) RMRs per process

- Maintain a tournament tree of locks.
- Processes compete to walk up tree.



Prior Results

→ $O(\log n)$ **(deterministic, tight)**

Yang and Anderson, "A fast, scalable mutual exclusion algorithm." 1995

→ $\Omega(\log n)$ **(deterministic, tight)**

Fan and Lynch, "An $\Omega(n \log n)$ lower bound on the cost of mutual exclusion." 2006

Attiya, Hendler, Woelfel, "Tight RMR lower bounds for mutual exclusion and other problems." 2008

→ $O(\log n / \log\log n)$ **(randomized, tight for adaptive?)**

Hendler and Woelfel, "Randomized mutual exclusion in $O(\log n / \log\log n)$ RMR." 2009

Our Results

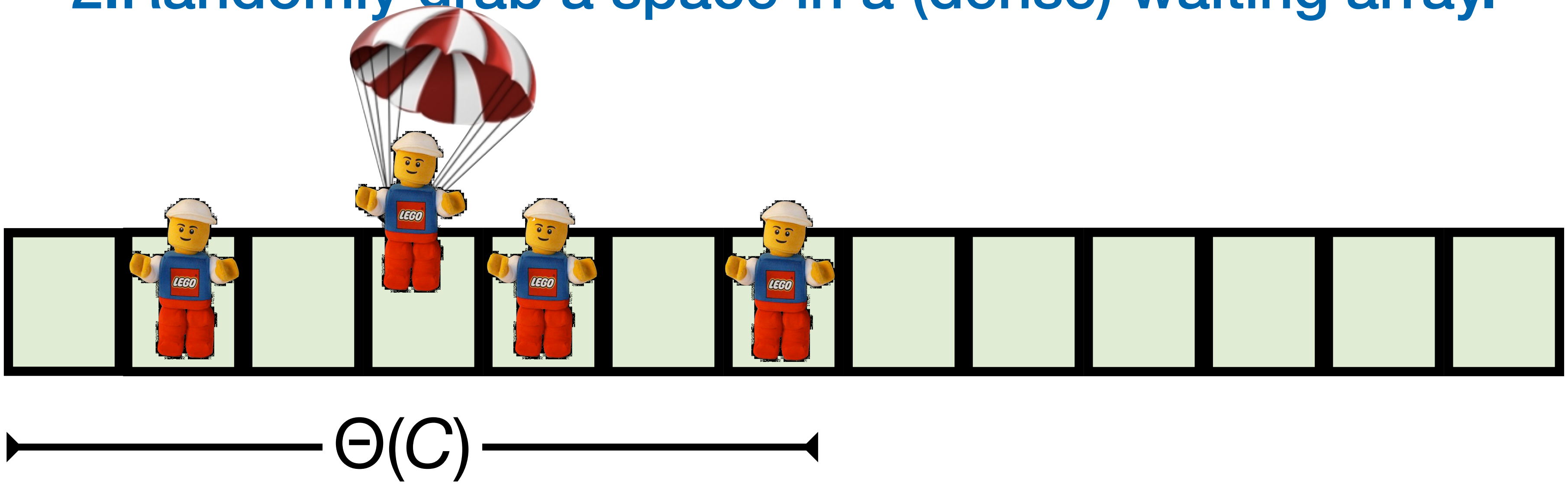
- New mutual exclusion algorithm with:
  - $O(\log^2 \log n)$ RMRs.
  - Randomized, subject to an oblivious adversary.
  - Each process enters the critical section whp.

- Incomparable with previous results because:
  - Weaker adversary: oblivious, not adaptive.
  - Liveness: guaranteed with high probability (instead of deterministic).

High-level Idea: Mutex ≈ Contention Resolution

Upon arriving:

1. Increment a process counter.
2. Randomly grab a space in a (dense) waiting array.

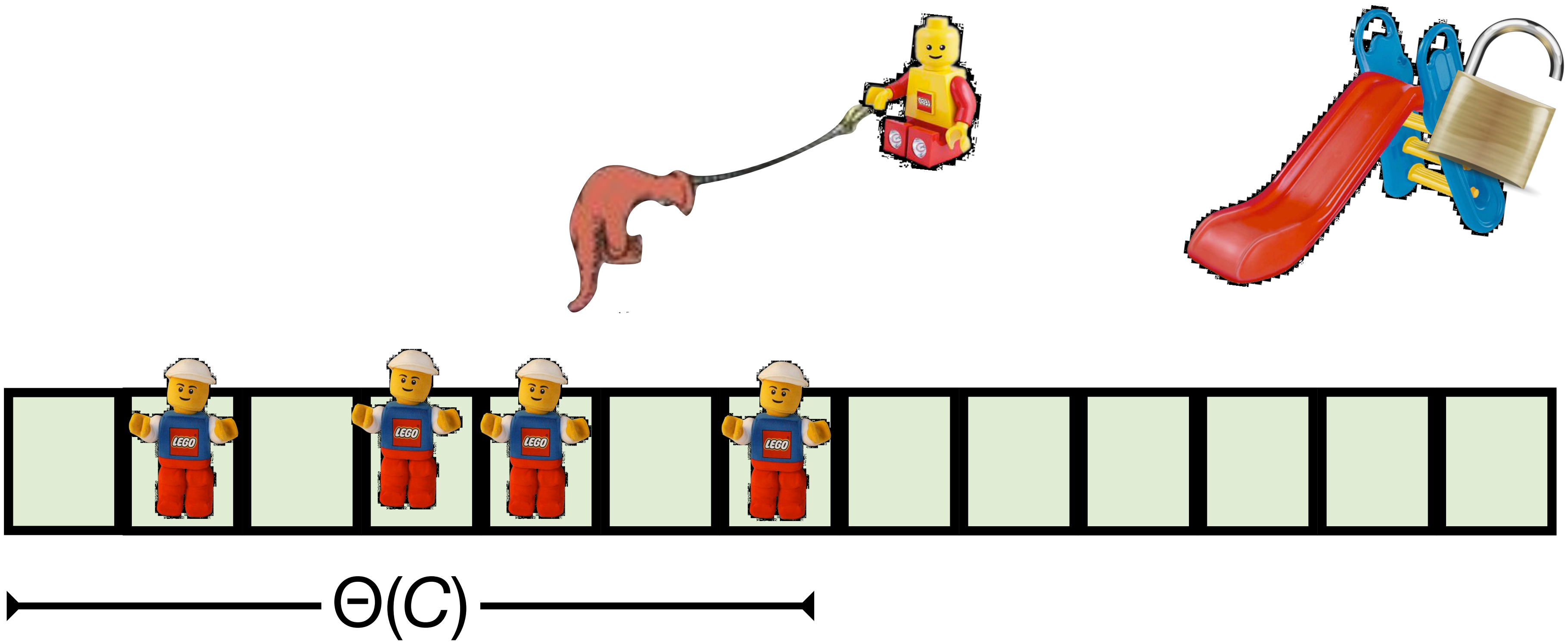


3. Try to grab mutex lock.
4. Sleep until awakened.

High-level Idea: Mutex ≈ Contention Resolution

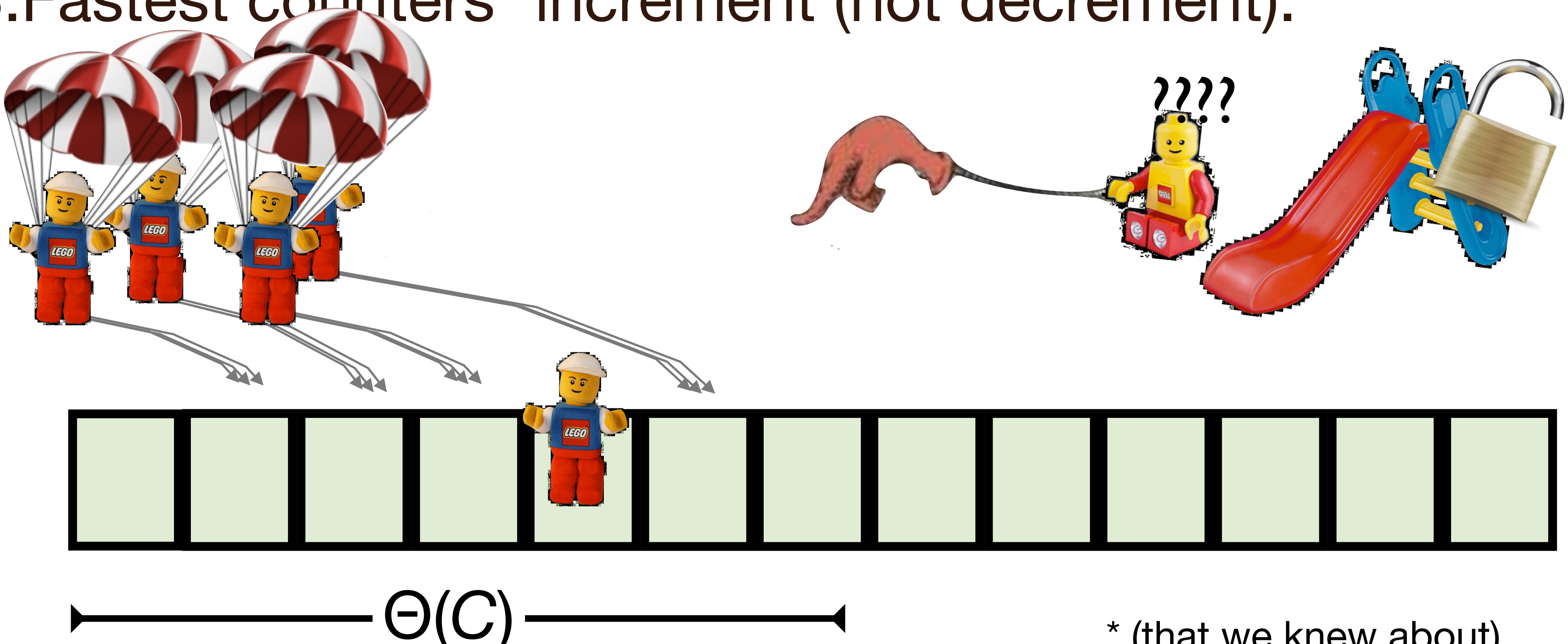
When departing:

1. Decrement the process counter.
2. Release the mutex lock.
3. Randomly pick a successor from the waiting array.



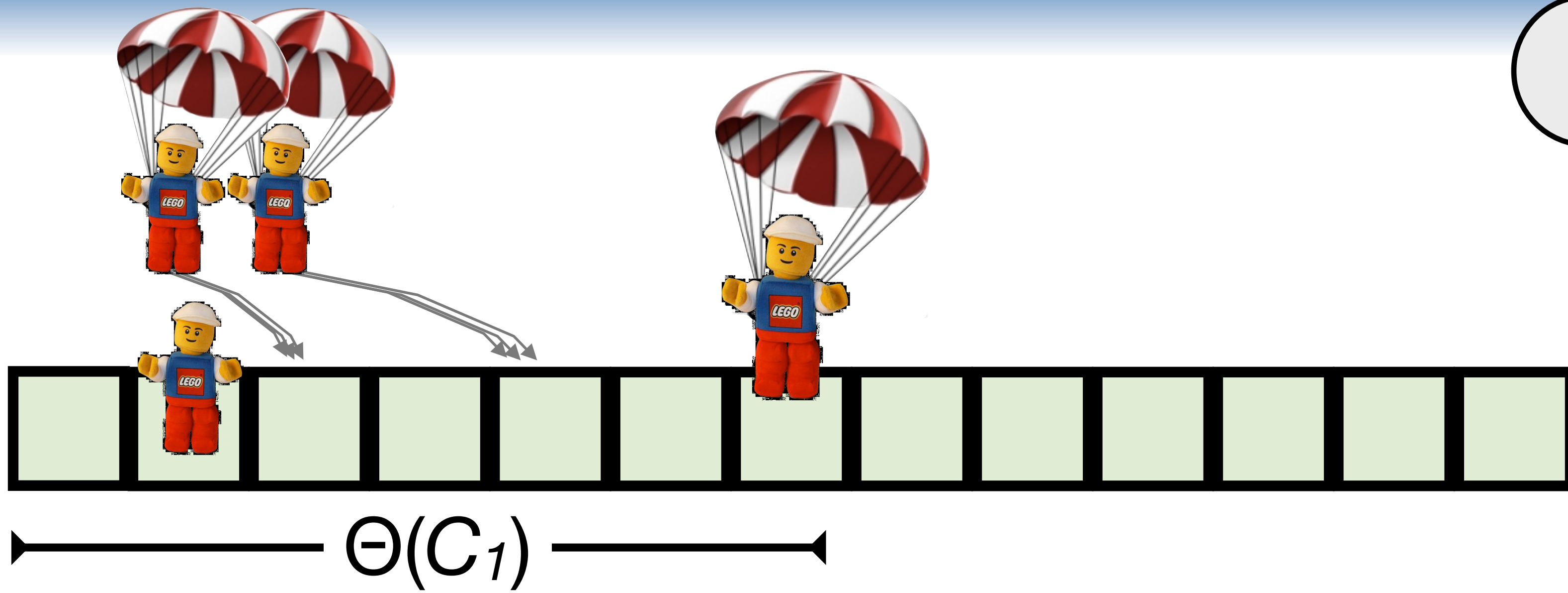
Problems with High-level Idea

1. The array could be sparse if many procs have “arrived” but not joined the array. Sparse array ⇒ slow to find successor.
2. Counting isn't cheap.
3. Fastest counters* increment (not decrement).

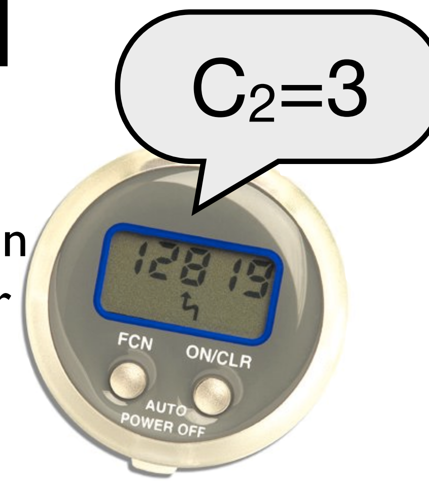


* (that we knew about)

(1) Dealing with Sparse Arrays



array-join
counter



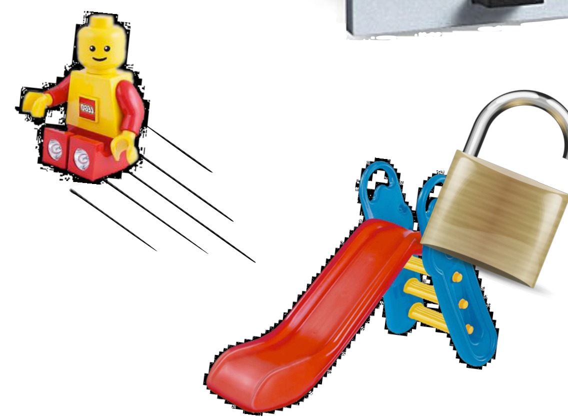
Upon arriving:

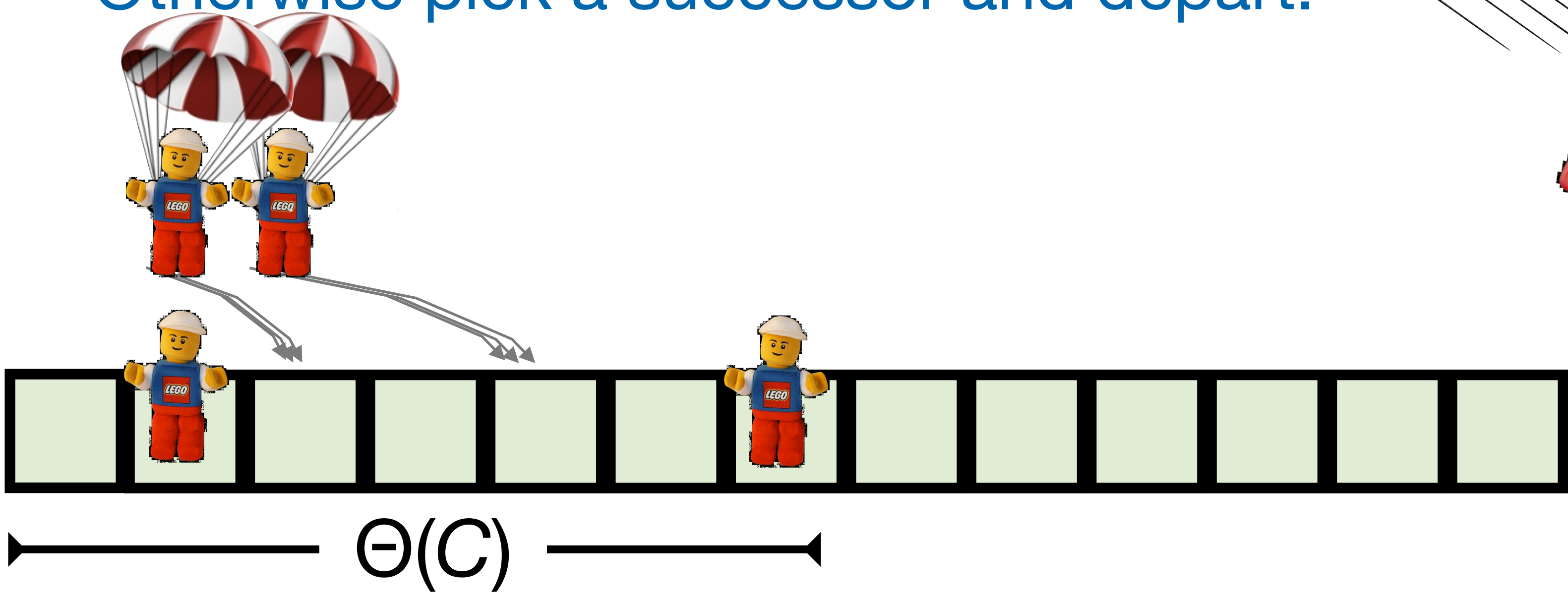
1. Increment process counter.
2. Randomly grab a space in a (dense) waiting array.
3. **Increment array-join counter.**
4. Try to grab mutex lock and then sleep.

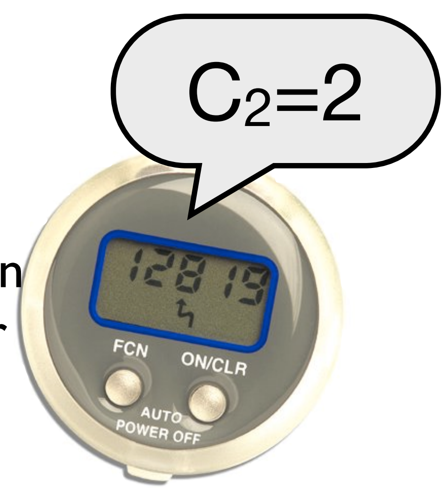
(1) Dealing with Sparse Arrays

When departing:

1. Decrement both counters.
2. Release the mutex lock.
3. **If $C_1>C_2/2$ then depart.**
   Otherwise pick a successor and depart.







array-join
counter

(2) Fast Approximate Counters

Approximate counting (standard trick):

- *increment*:
  - Choose **cnt** with probability $\mathbf{1/2^{cnt}}$
  - Write **cnt** to max-register
- read:
  - **cnt** = read-max-register
  - return $\mathbf{2^{cnt}}$.

max value = log(n)
Cost: O(loglog n) [AAC'09]

- Example: 64 processes
  - With constant probability, at least one process chooses 6
  - With constant probability, no process chooses a value larger than 6
  - => With constant probability, counter returns 64.
  - => Constant factor approximation

(2) Fast Approximate Counters

Approximate counting (standard trick):

- State:
  - Bounded counters C[1], C[2], C[3], ..., C[log n]
  - max-register: max-value log(n)

- *increment*:
  - Choose **cnt** with probability **$1/2^{cnt}$**
  - Increment counter **C[cnt]**.
  - If **C[cnt]** > log(n), then write **cnt** to max-register

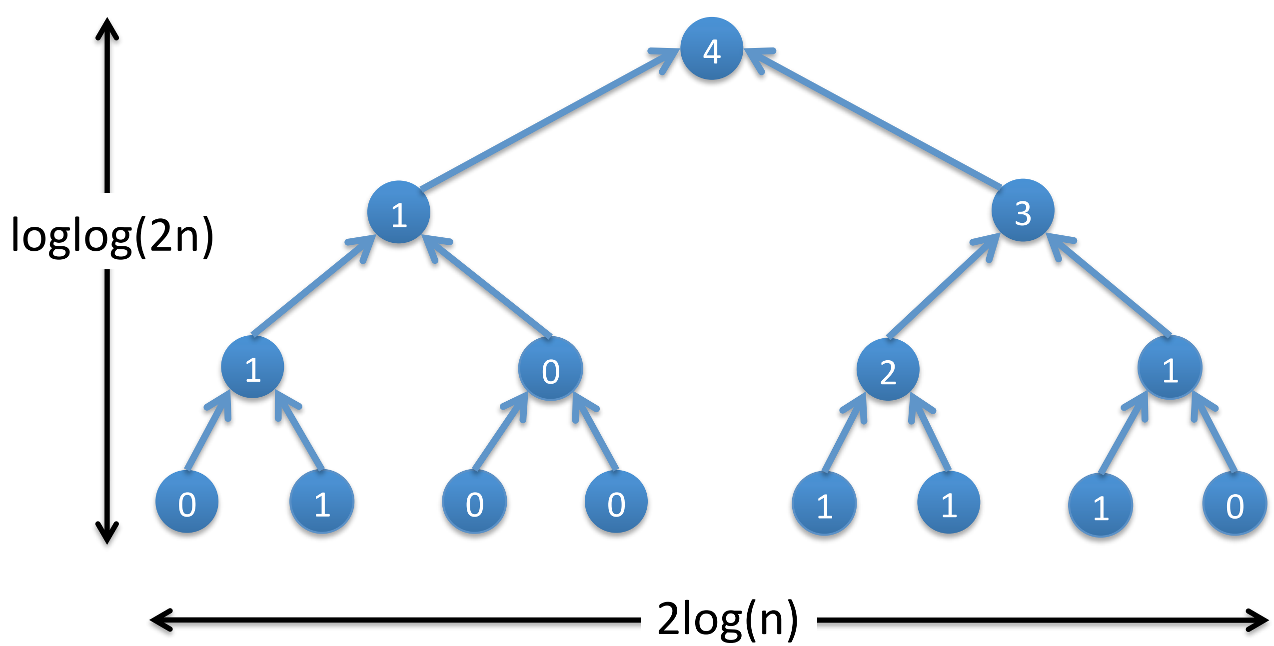
- read:
  - **cnt** = read-max-register
  - return **$\log(n) \cdot 2^{cnt}$**.

(2) Fast Approximate Counters

Approximate counting (standard trick):

- Cost of [AAC'09] counter: O(log n*loglog n)
  - One leaf per process

- Small tweak: 2log(n) leaves total
  - On increment, choose a random leaf



(2) Fast Approximate Counters

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- read:
  - **cnt** = read-max-register
  - return **$\log(n)*2^{cnt}$**.

Cost: O(loglog n)

(2) Fast Approximate Counters

Approximate counting (standard trick):

- If (> log n) increments) then return value is a constant-factor approximation with high probability.

- What about small values?
  - Use small-counter with max-value log(n).
  - Increment small-counter and ...
  - Read both small-counter and ...

(2) Fast Approximate Counters

Approximate counting (standard trick):

- State:
  - Bounded counters C[1], C[2], C[3], ..., C[log n]
  - max-register: max-value log(n)

- *increment*:
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  - Increment counter **C[cnt]**.
  - If **C[cnt]** > log(n), then write **cnt** to max-register

- read:
  - **cnt** = read-max-register
  - return **$\log(n)*2^{cnt}$**.

Constant-factor approximation, whp

(3) Counters that Increment/Decrement

Replace process counter with two counters.

process counter

≈

arrival counter
(increments)

-

departure counter
(decrements)

(3) Counters that Increment/Decrement

Replace process counter with two counters.

≈

-

process counter

arrival counter
(increments)

departure counter
(decrements)

Problem: Does this still work, since we use approximate counters?

?
≈

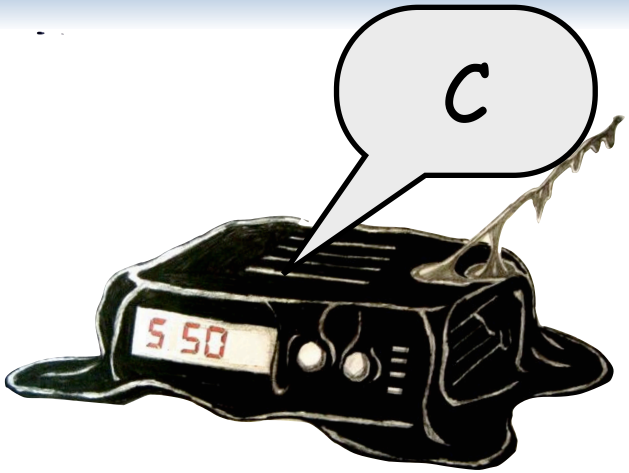
-

approx process counter

arrival counter
(approx after poylog n)

departure counter
(exact)

(3) Counters that Increment/Decrement



approx process counter

$\stackrel{?}{\approx}$

approx arrival counter (increments)

-

exact departure counter (decrements)

Good approximation when:

- $C_{arrive} > 2\ C_{leave}$.
- C_{arrive}, C_{leave} = polylog n.

(3) Counters that Increment/Decrement

approx process counter

approx arrival counter (increments)

exact departure counter (decrements)

Good approximation when:

- $C_{arrive} > 2\ C_{leave}$.
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Poor approximation otherwise.

(3) Counters that Increment/Decrement

approx process counter

approx arrival counter (increments)

exact departure counter (decrements)

Good approximation when:

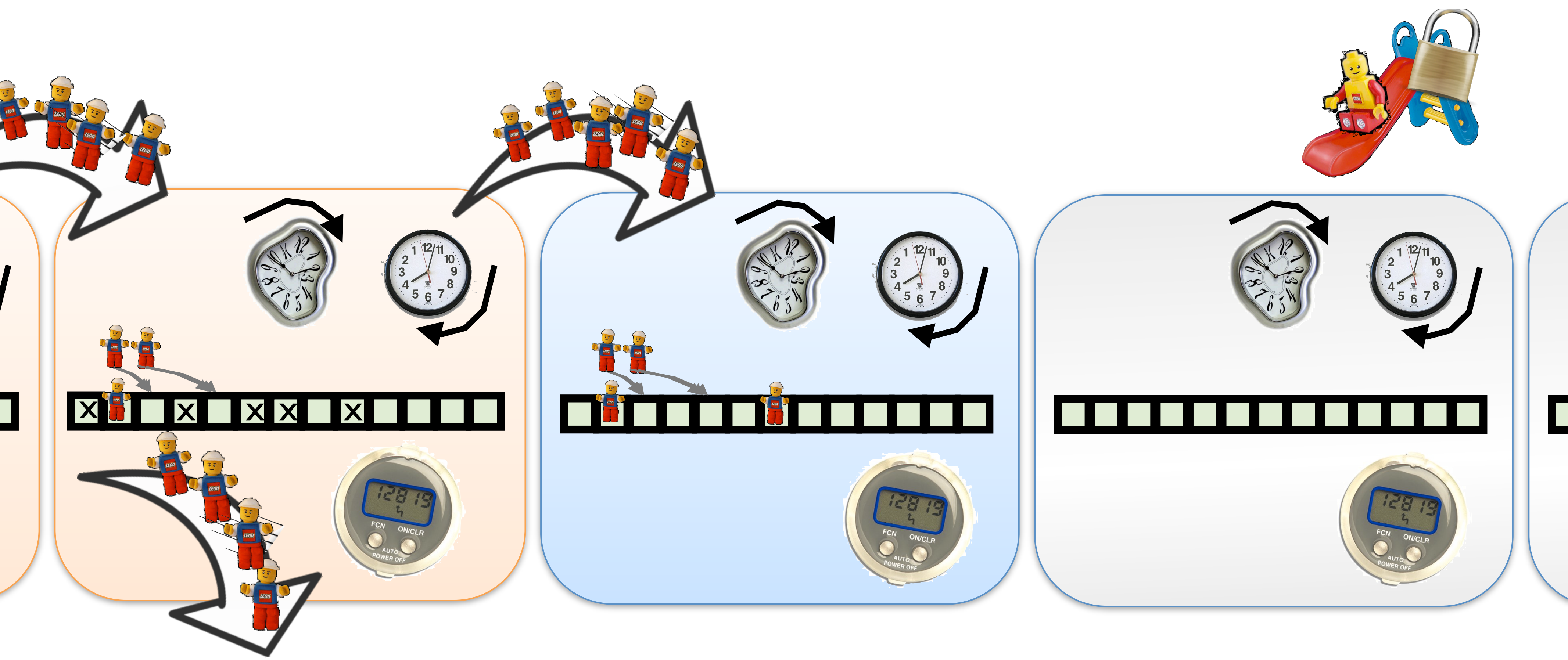
- $C_{arrive} > 2\ C_{leave}$.
- C_{arrive}, C_{leave} = polylog n.

Poor approximation otherwise.

When the approximation gets bad... reset counter.

Computation Proceeds in Epochs

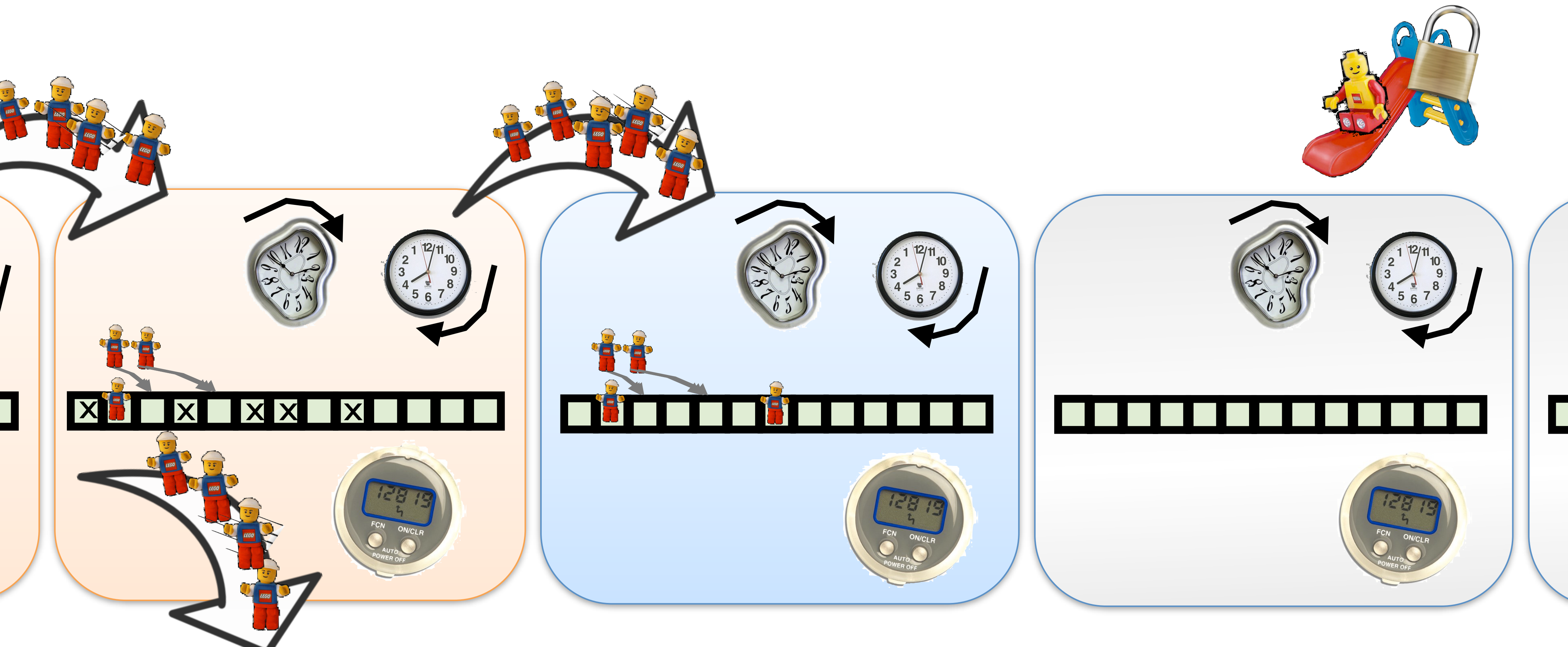
Each epoch uses a new copy of the data structure.



Computation Proceeds in Epochs

Each epoch uses a new copy of the data structure.

- An O(1)-fraction of procs finish in each epoch.
- The remaining procs are kicked out of the waiting array. These join the waiting array of next epoch.



Computation Proceeds in Epochs

The cost to rejoin a waiting array is amortized against the procs that complete in the epoch.

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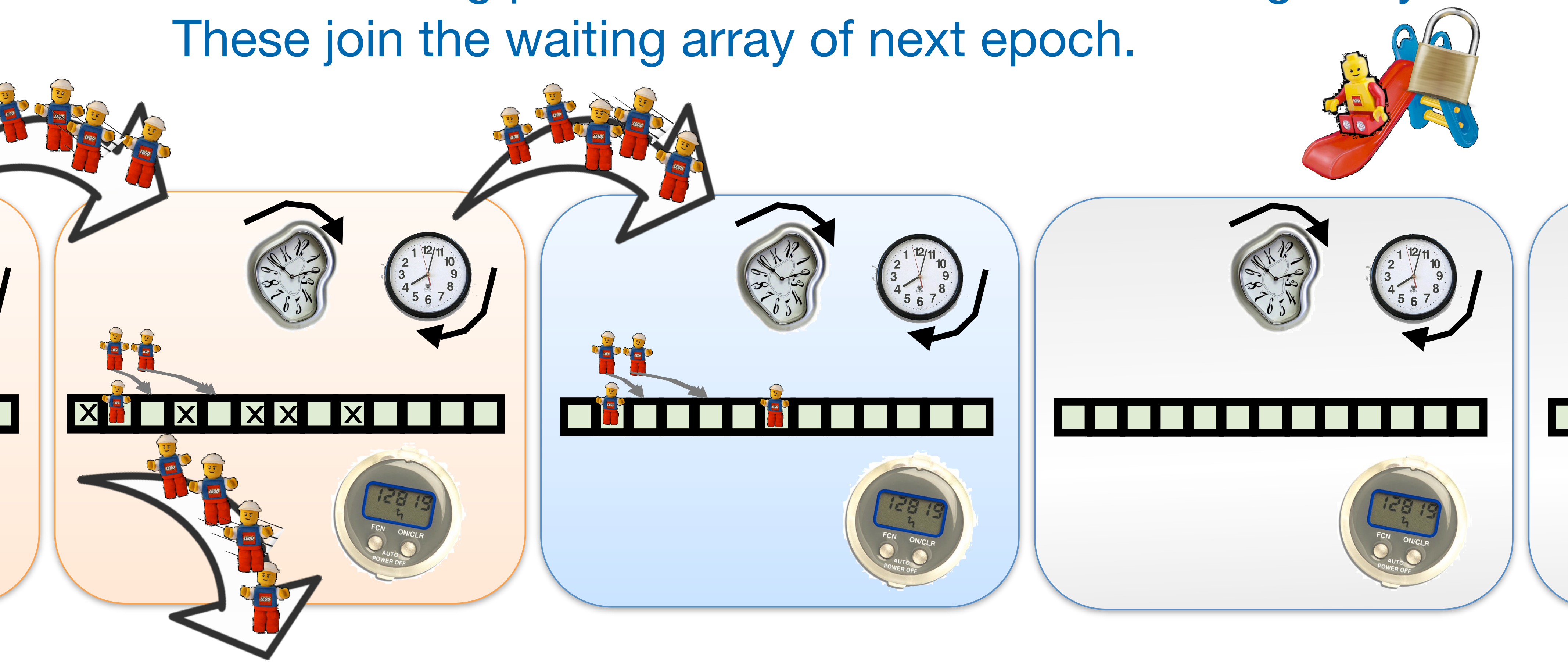
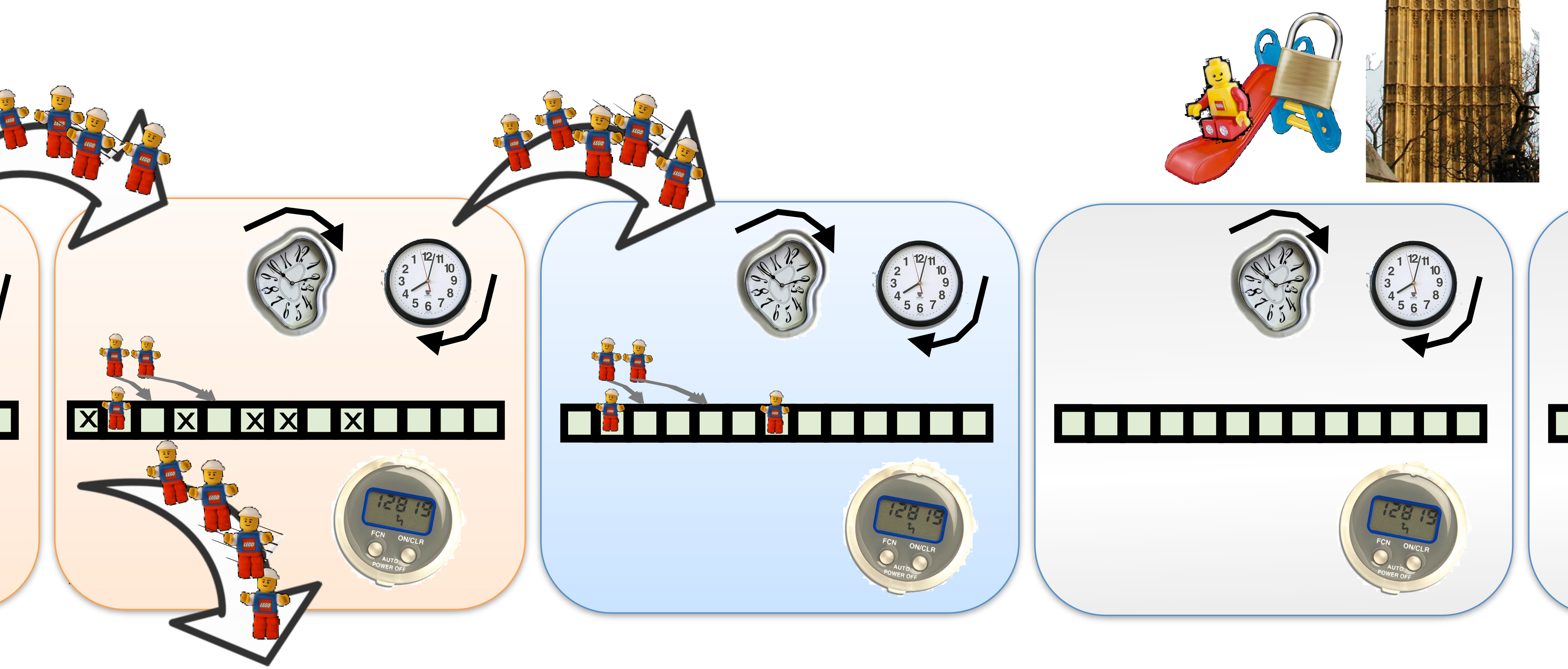


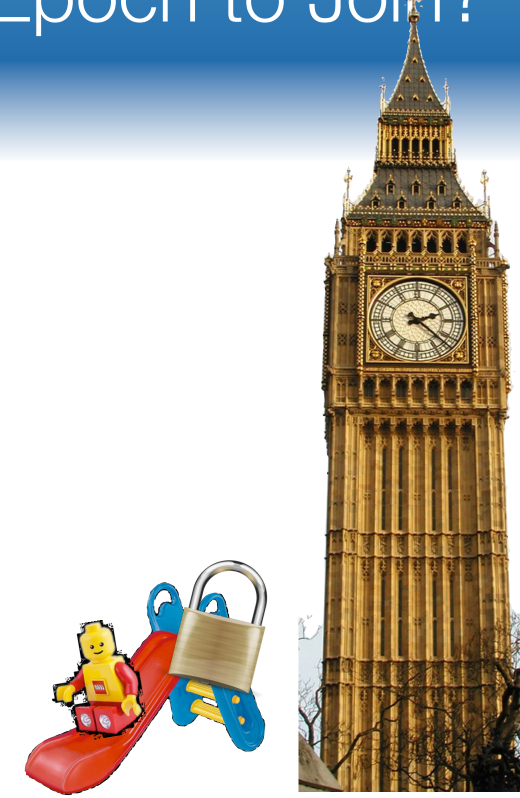
How Does a Process Determine Which Epoch to Join?

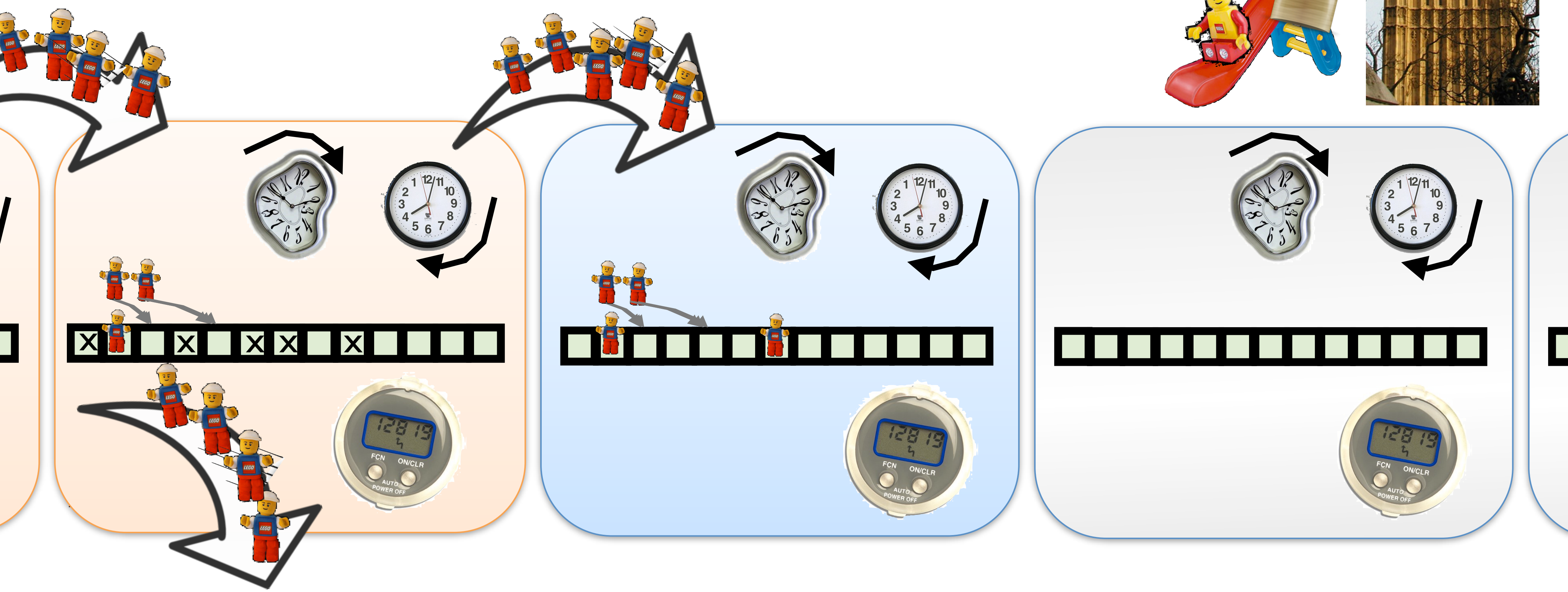
An epoch counter.



How Does a Process Determine Which Epoch to Join?

So a process arrives, reads, the counter, and joins current epoch.

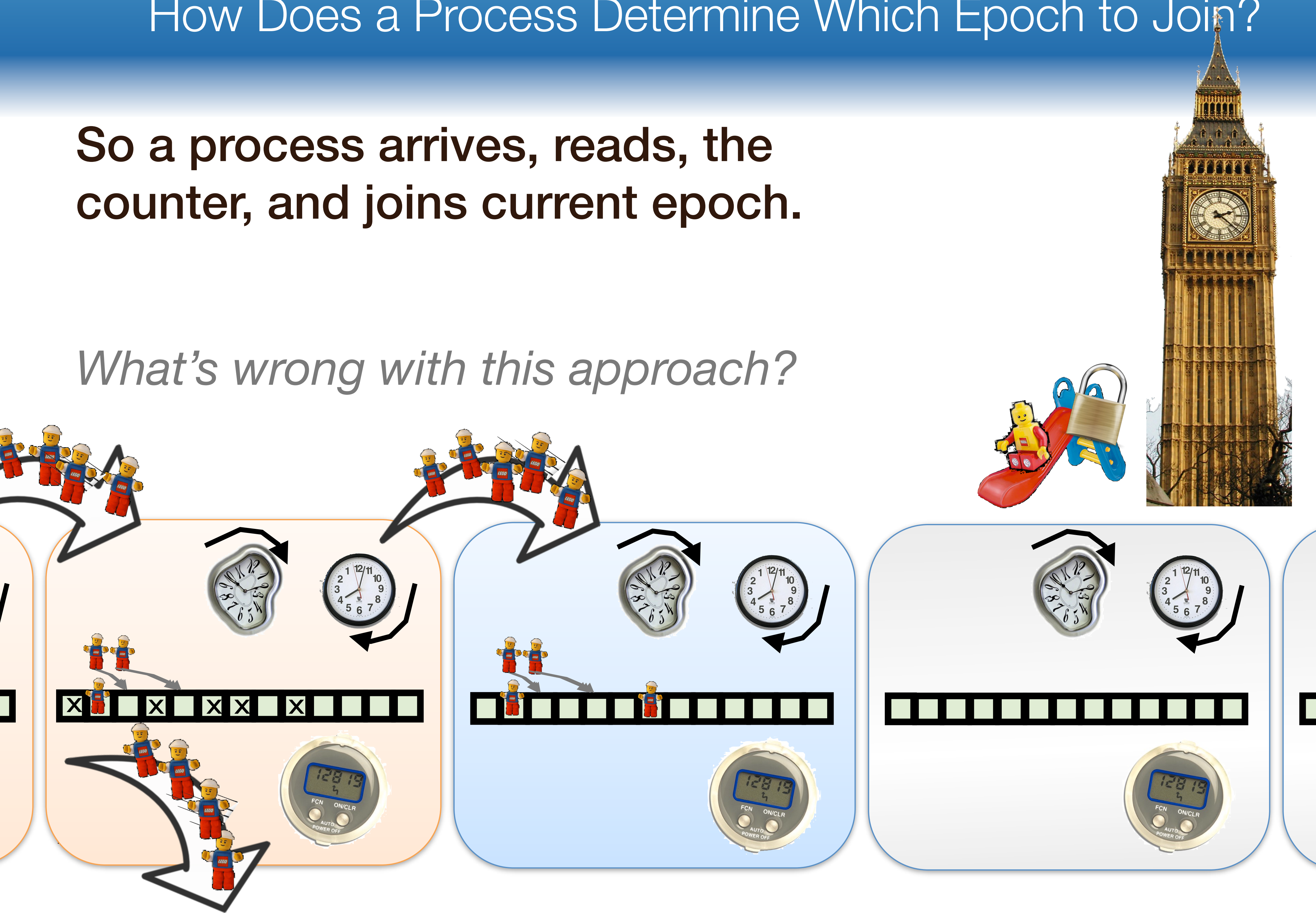




How Does a Process Determine Which Epoch to Join?

So a process arrives, reads, the counter, and joins current epoch.

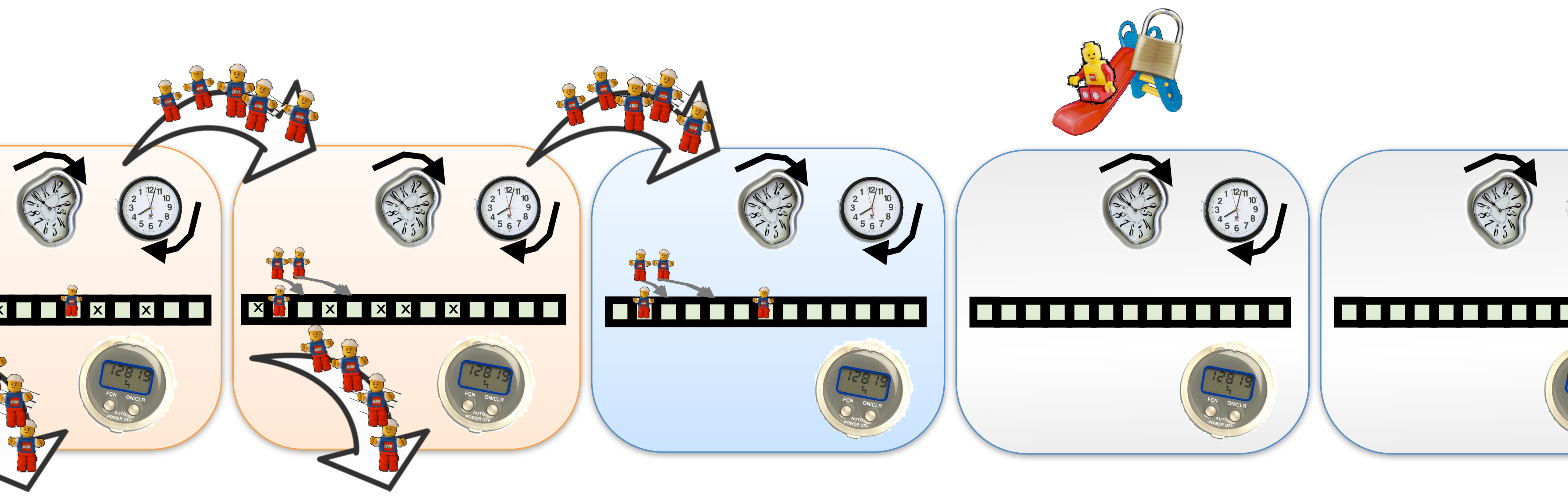
What's wrong with this approach?



How Does a Process Determine Which Epoch to Join?

Problem: what amortizes cost to read counter?

- Θ(n) procs may read the counter in each epoch.
  - Proc reads epoch counter. Falls asleep.
  - Discovers the epoch has changed.
  - Reads the counter again. Falls asleep again.
  - Etc



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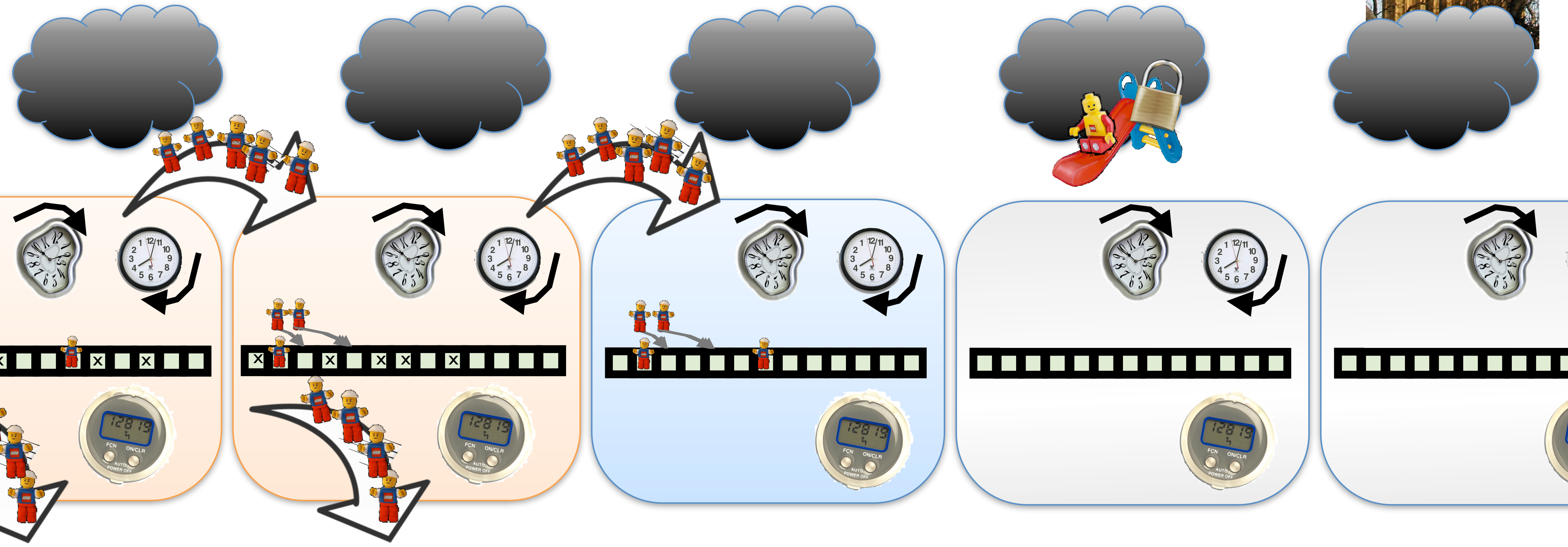
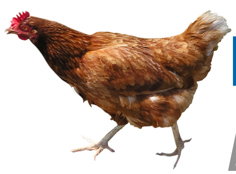


Chicken-and-Egg Problem with Epoch Counter

Which comes first?

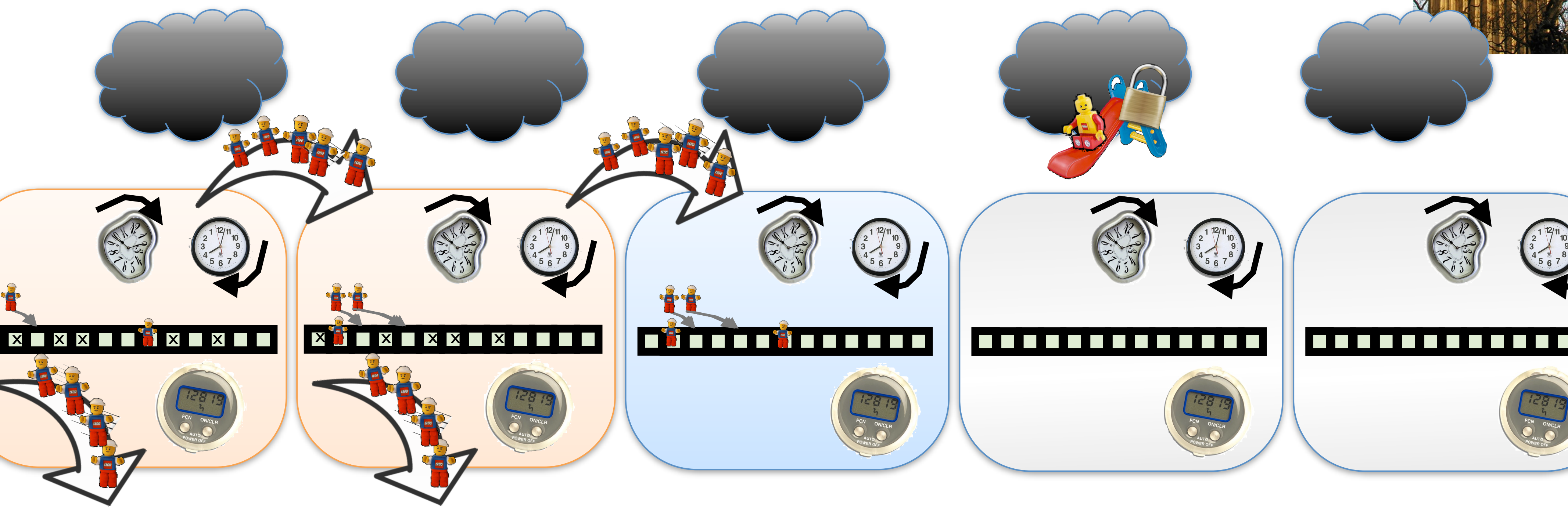


Read the epoch number.

But this read isn't amortized.

Increment some process counter.

But which one? We don't know the epoch.



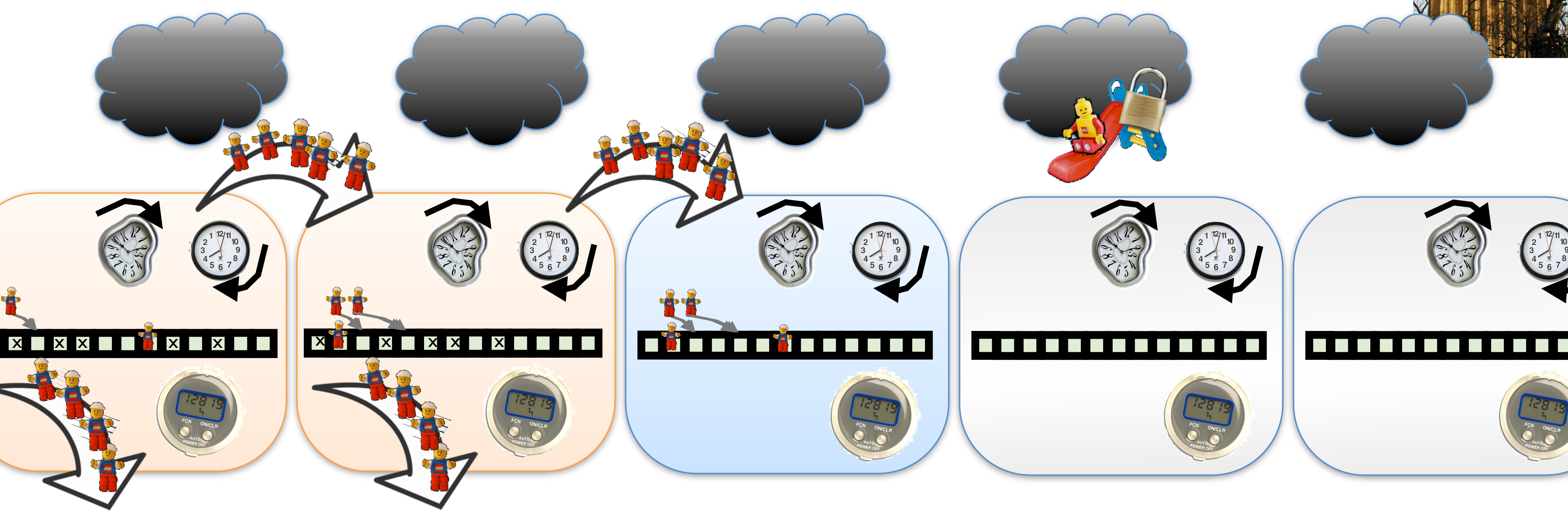
Chicken-and-Egg Problem with Epoch Counter

Chickens are fairly recent. E.g., 10s of millions of years. Eggs have been around for >400 Million years.

Chicken-and-Egg Problem with Epoch Counter

When a process arrives, its first operation must be a write.

- Proc must write even without knowing epoch #.
- The write must be visible to other procs in the epoch.



Chicken/Egg Problem Resolved

A process's first operation must be a write.

This write must increment the population count.

Chicken/Egg Problem Resolved

A process's first operation must be a write.
This write must increment the population count.

Idea: there isn't one arrival counter per phase.
There are only 3. These are reused.

For epochs
1,4,7,10,....

For epochs
2,5,8,11,....

For epochs
3,6,9,12,....

Chicken/Egg Problem Resolved

Which counter should the proc increment?

Answer: choose one randomly.

Recall:

- writing one bit in a random (not uniform) location is enough to record one's presence.
- Once this bit is written, the procs can read the epoch counter and proceed as before.....

For epochs
1,4,7,10,....

For epochs
2,5,8,11,....

For epochs
3,6,9,12,....

Chicken/Egg Problem Resolved

Resetting the counter.

- When the phase ends, reset the counter.
- This reset is not atomic (cannot use pointer swings).

For epochs
1,4,7,10,....

For epochs
2,5,8,11,....

For epochs
3,6,9,12,....

Conclusion

Trend: a distributed realization that many classic problems have sublogarithmic solutions.

- O(log n/loglog n) mutex [Hendler and Woelfel 09]
- O(log* n) test ‘n’ set (George’s talk)
- O(loglog n) consensus (Jim’s talk)
- O(polyloglog) mutual exclusion (this talk)

We are collectively discovering what can/can’t be done in sublogarithmic time.

- Most of these results have oblivious adversaries.
- Yes: approximate counting, leader election.
- No: exact counting, random sampling.

Conclusion

Our algorithm is composed of building blocks.

- Counters, approximate counters, max registers, arrays, CAS, etc.

Alas, most of these aren't strongly linearizable.

- Even when they are (e.g., CAS), it's not not relevant to an oblivious adversaries.

Result:

- Inelegant proofs.
- Lisa, Philipp, Wojciech, George, please hurry up :-)

Open Questions

Stronger adversary?

Adaptive to number of participants?

- This paper: running time depends on n.

Monte Carlo vs. Las Vegas?

- This paper: No deadlock with high probability

Lower bounds?

Alternative constructions that are simpler?